

CBSE | CUET | JEE

www.mtg.in
December 2022
Pages 80 | ₹ 40

MATHEMATICS

today

India's #1
MATHEMATICS MONTHLY FOR
CBSE, CUET, JEE & OTHER PET EXAMS

ARE YOU READY FOR
OLYMPIADS ?

SOF
SOLVED OLYMPIAD FOUNDATION

IMO
INDIAN MATHEMATICS OLYMPIAD

Class
XI-XII

see inside

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

Class
XI & XII

CBSE

warm up!

BEAT THE TIME TRAP

CELEBRATING
40
YEARS OF
EXCELLENCE

mtg

Trust of more than
1 Crore Readers
Since 1982



2022120013538

Class
XI & XII

MONTHLY TEST DRIVE

IMPORTANT

Prices set to
increase in January
Subscribe today at
current prices for
max savings

Go to
page 76
now

mtg

2022-23

Based on
Competency Based
Education (CBE)



100
PERCENT

Science, Mathematics,
English & Social Science

Physics, Chemistry,
Mathematics & Biology

THE SMART STUDY SYSTEM

100%
CBSE

MTG's 100 Percent Series is based on the most current CBSE guidelines and curriculum

100%
SCIENTIFIC

MTG's editorial team knows what it takes to score more in exams - it keeps a close watch on changing examination patterns and has been able to put all ingredients critical to success together in a package aptly titled 100 Percent

100%
CONTENT

Be it easy-to-comprehend text, graphics, illustrations or concept maps that complement, understanding theory and concepts is so straightforward with 100 Percent

100%
CHECKS

After each topic, in every chapter, 100 Percent presents students with a variety of Q&A, including "Try Yourself", "NCERT Focus" & "CBSE Focus"

100%
PRACTICE

100 Percent goes all out to ensure students are prepared for a diverse set of future challenges through a variety like Competition Focus, Exam Drills and Practice Papers, even Viva Voce Q&A for lab-based experiments

Series
Available
For CBSE
Classes
9 to 12



SCAN QR CODE



Available at all leading booksellers. To buy online, Scan QR code
For more information, call 0124-6601200 (Lines open Mon to Sat, 9 am to 5:30 pm).
Or e-mail at info@mtg.in now.

Disclaimer : The content available in MTG digital/physical books is intellectual property of MTG Learning Media Pvt Ltd. Paid content on this website after purchase is not entitled for free distribution on whatsapp, telegram, twitter, facebook, and all other social media and digital channels. The registered owner/admin/ individual of any digital channel will be liable for an infringement of the copyright for illegally using the material / copyrighted content of MTG Learning Media Pvt Ltd.

CBSE

BOARD EXAM 2022-23

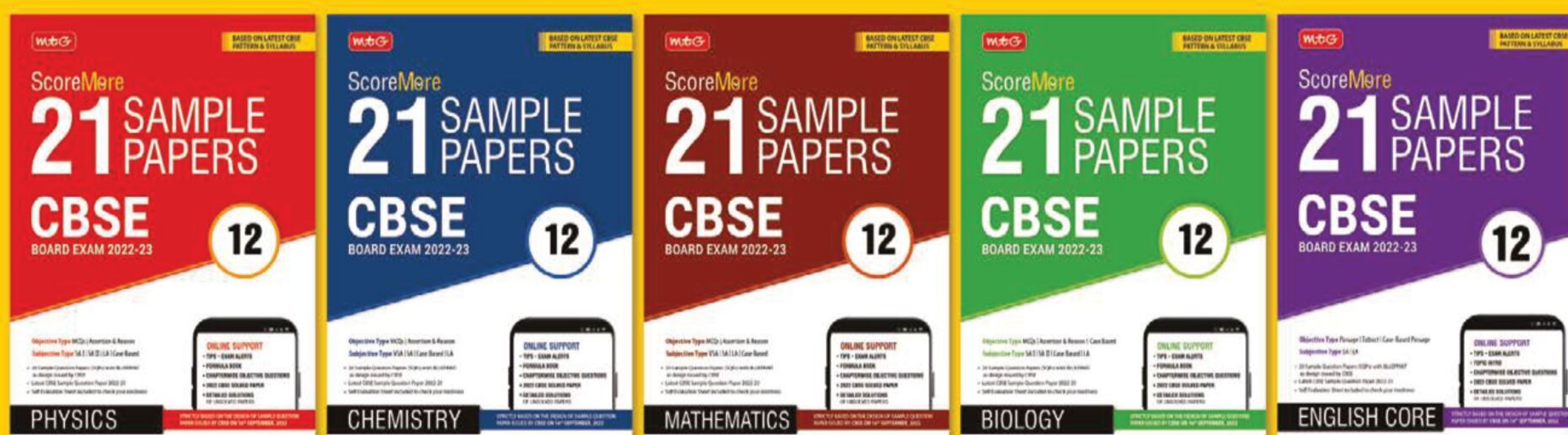
Strictly Based on the design of
Sample Question Paper issued by
CBSE on 16th September 2022

12



**BASED ON LATEST CBSE
PATTERN & SYLLABUS**

**Price
₹ 300/-
each**

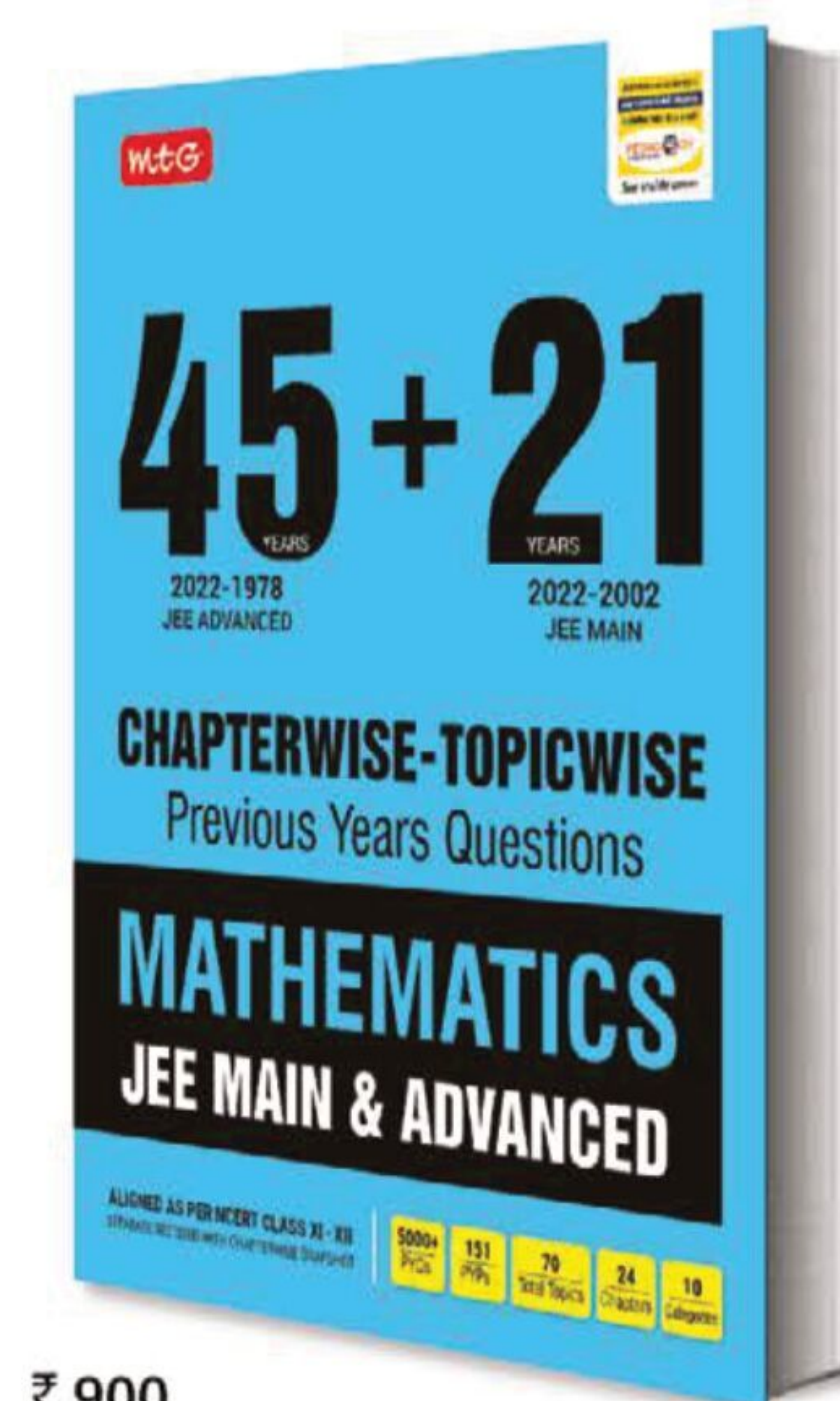
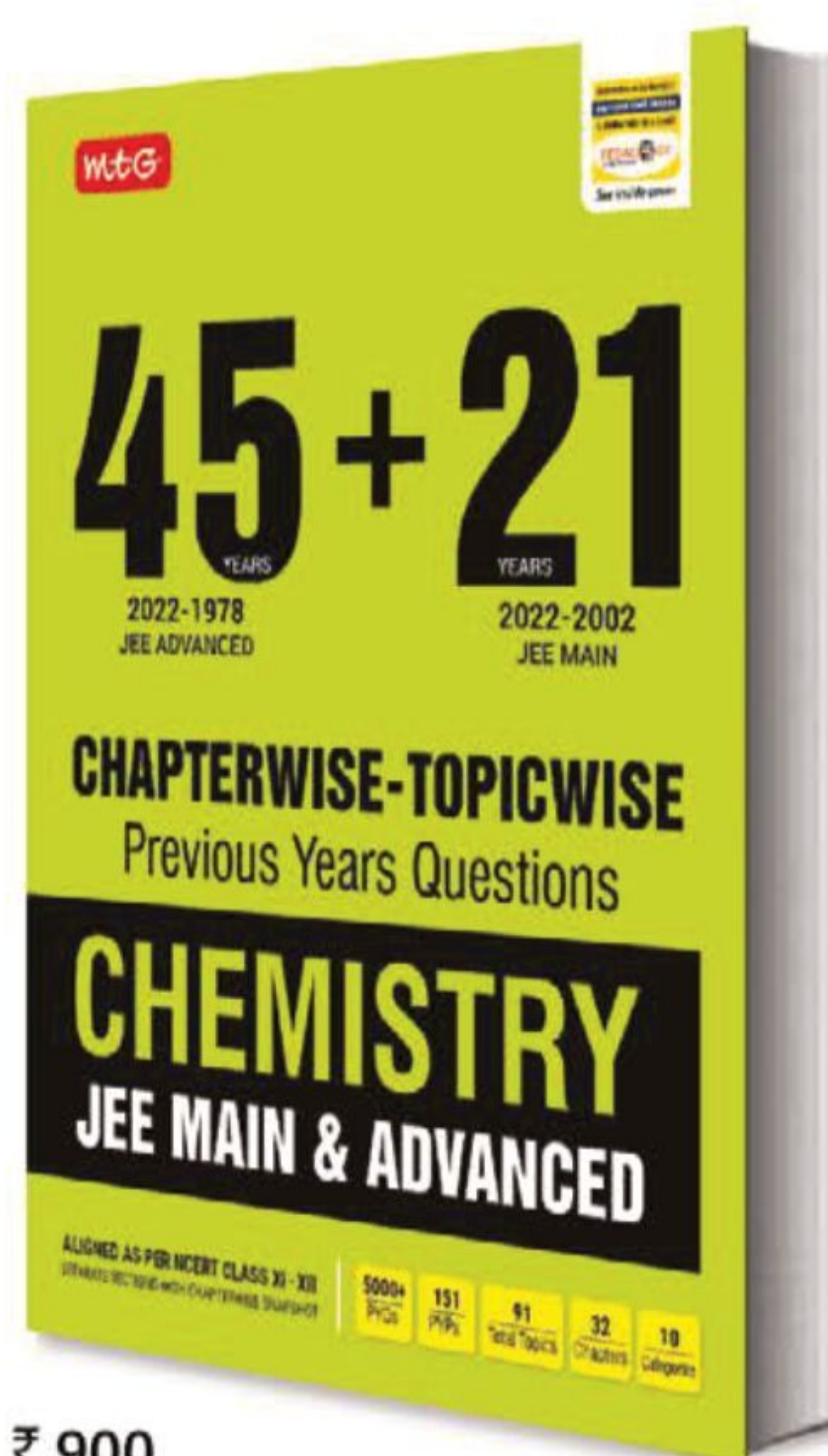
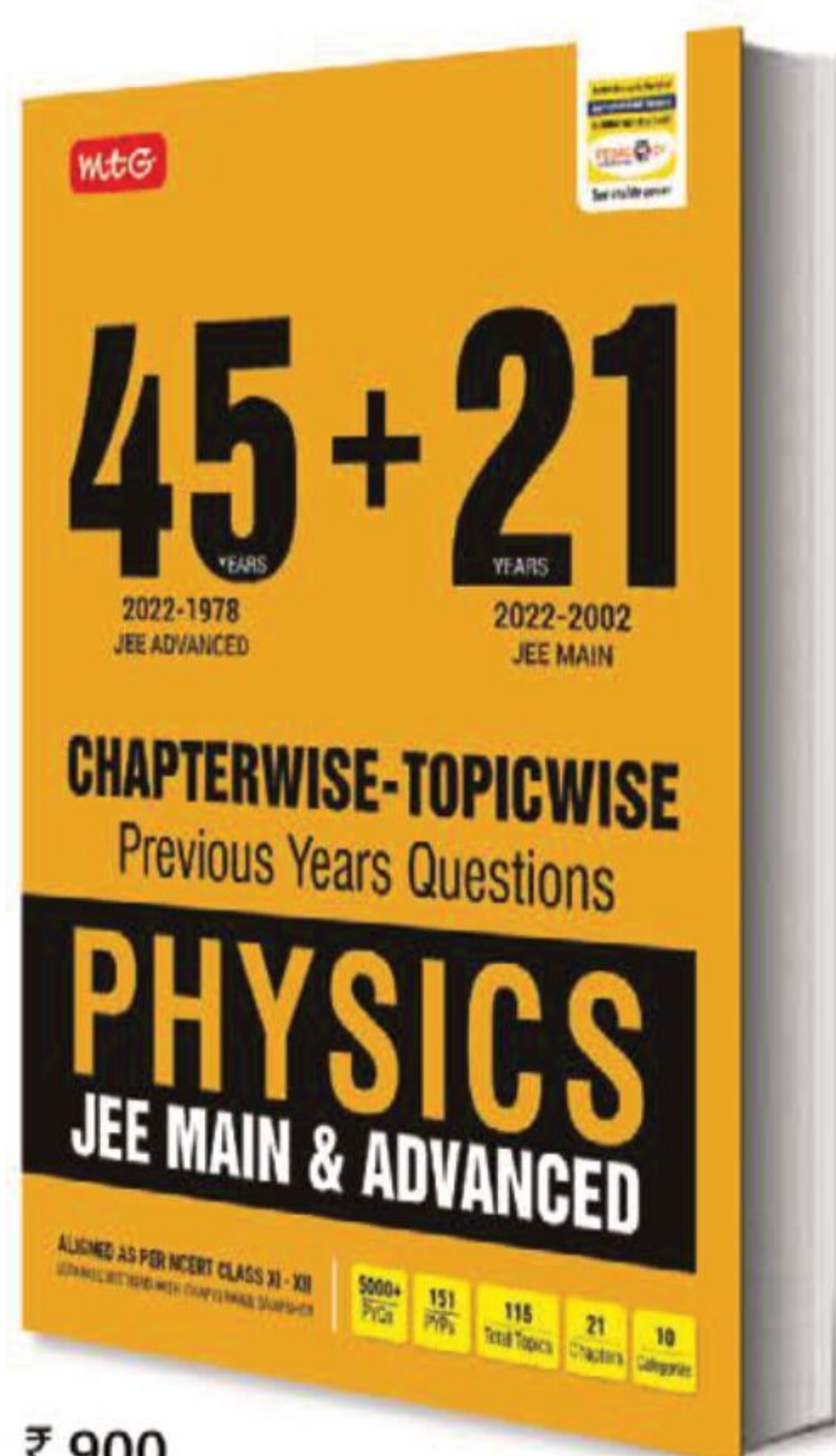


Available at all leading book shops throughout India.
For more information or for help in placing your order, Call 0124-6601200 or e-mail: info@mtg.in



Some of the best lessons are learnt from history!

Introducing Chapterwise-Topicwise Solutions to JEE.
Crack the past to see if you will last.



Tune. Fine tune. Reach the peak of your readiness for JEE with MTG's 45+21 Years Chapterwise-Topicwise Solutions. It is undoubtedly the most comprehensive 'real' question bank, complete with detailed solutions by experts.

Studies have shown that successful JEE aspirants begin by familiarising themselves with the problems that have appeared in past JEEs as early as 2 years in advance.

Making it one of the key ingredients for their success. How about you then?
Get 45+21 Years Chapterwise-Topicwise Solutions to start your rehearsals early.



Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Visit www.mtg.in to buy online!
Call 0124-6601200 or e-mail: info@mtg.in



Scan to explore & buy more
JEE Books on mtg.in



MTG has been sincerely serving to brilliant students, compassionate teachers and other educational prodigies through its unparalleled magazines since 1982. Now, in order to come at par with the constantly developing and dynamic market, MTG plans to upgrade its magazines. Keeping in mind the inflation faced by one and all, and price rise of even basic commodities MTG is forced to revise the price of magazines from ₹ 40 to ₹ 75 from January 2023. We deeply expect your support and acumen for the same. To enhance the value of magazines more pages and more features are added, like:

- Update yourself with latest researches
- Equip yourself with unique career options
- Get latest exam updates
- Get to know the selection procedure of various universities offering different courses
- Get a peek into monthly current affairs
- Know your IQ level with various quizzes

MATHEMATICS today

Vol. XL

No. 12

December 2022

Managing Editor
Mahabir Singh

Editor
Anil Ahlawat

Corporate Office:

Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in

Regd. Office:

406, Taj Apartment, Near Safdarjung Hospital, New Delhi - 110029.

CONTENTS

Competition Edge

CUET (UG) Chapterwise Practice Paper 2023 7

Integrals / Applications of Integrals

Challenging Problems 34

Beat the Time Trap 42

You Ask We Answer 46

Class 11

Are you ready for Olympiads? 21

Concept Map 40

Complex Numbers

CBSE Warm Up! 47

Conic Sections

Monthly Test Drive 57

Binomial Theorem

Class 12

Are you ready for Olympiads? 27

Concept Map 41

Three Dimensional Geometry (Line)

CBSE Warm Up! 60

Differential Equations

Monthly Test Drive 74

Application of Derivatives

Subscribe online at www.mtg.in

Individual Subscription Rates	9 months			Combined Subscription Rates	9 months		
	15 months	27 months			15 months	27 months	
Mathematics Today	300	500	850	PCM	900	1400	2500
Chemistry Today	300	500	850	PCB	900	1400	2500
Physics For You	300	500	850	PCMB	1200	1900	3400
Biology Today	300	500	850				

Send D.D./M.O in favour of MTG Learning Media (P) Ltd.
Payments should be made directly to : MTG Learning Media (P) Ltd,
Plot No. 99, Sector 44, Gurugram - 122003 (Haryana)
We have not appointed any subscription agent.

Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.

Editor : Anil Ahlawat

Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/ Infocus features are marketing incentives. MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.

Copyright© MTG Learning Media (P) Ltd.

All rights reserved. Reproduction in any form is prohibited.



Success Guaranteed in CUET (UG)

Common University Entrance Test Under-Graduate Programmes
with **MTG CUET (UG) Practice Papers**

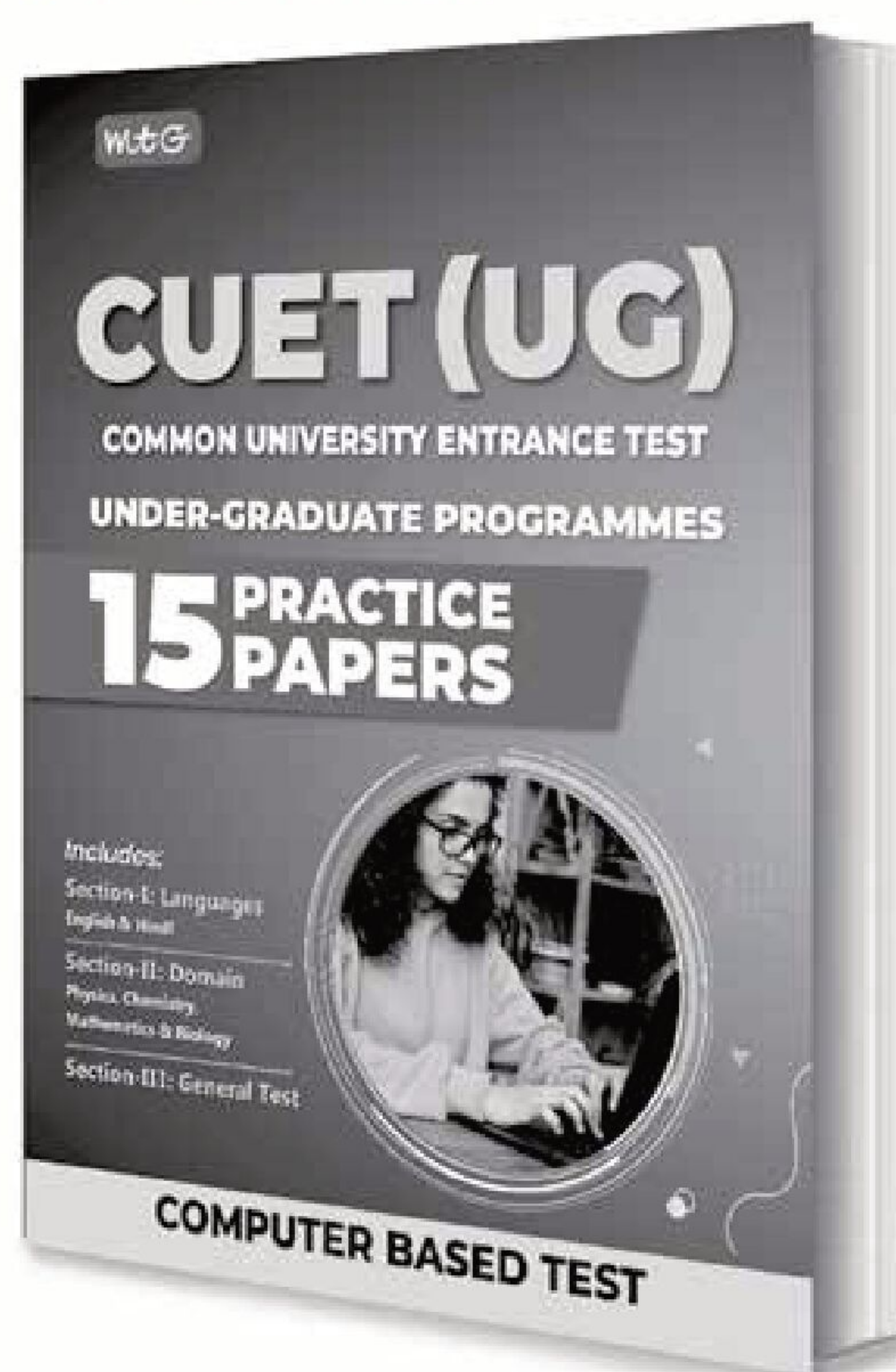
From the academic session 2022-23, admissions to Undergraduate Programmes in all central universities in the country will take place through Central University Entrance Test (CUET).

To help students with their preparation for the above test MTG is coming up with the book CUET 2023 which caters the Science Stream for the test. The MTG CUET (UG) – 2023 title covers 15 Practice papers which include following subjects in each practice paper :

Section IA Languages : English and Hindi

Section II Domain : Physics, Chemistry, Mathematics and Biology

Section III General Test : General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning, Logical and Analytical Reasoning



Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Visit www.mtg.in to buy online!
Call 0124-6601200 or e-mail: info@mtg.in

Visit
www.mtg.in
for latest offers
and to buy
online!

CHAPTERWISE PRACTICE PAPER 2023

CUET (UG)

SERIES - IV Integrals | Applications of Integrals

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time Allowed : 45 Minutes

Maximum Marks : 200

Multiple Choice Questions (MCQs)

1. Evaluate :

$$\int \left\{ (2x-3)^5 + \frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}} + \frac{1}{2-3x} + \sqrt{3x+2} \right\} dx$$

(a) $\frac{1}{12}(2x-3)^6 - \frac{1}{14}(7x-5)^{-2} + \frac{2}{5}\sqrt{5x-4} - \frac{1}{3}\log|2-3x| + \frac{2}{9}(3x+2)^{3/2} + C$

(b) $\frac{1}{12}(2x-3)^6 - \frac{1}{14}(7x-5)^{-2} + \frac{2}{5}\sqrt{5x-4} + C$

(c) $\frac{1}{12}(2x-3)^4 + \frac{1}{14} + (7x-5)^{-2} + C$

(d) None of these

2. $\int xe^{x^2} dx$ is equal to

(a) $-\frac{e^{x^2}}{2} + C$

(b) $\frac{e^{x^2}}{2} + C$

(c) $\frac{e^x}{2} + C$

(d) $-\frac{e^x}{2} + C$

3. Evaluate : $\int \frac{\sin 2x}{\sin\left(x - \frac{\pi}{3}\right)\sin\left(x + \frac{\pi}{3}\right)} dx$

(a) $\log\left|\sin\left(x + \frac{\pi}{3}\right)\right| - \log\left|\sin\left(x - \frac{\pi}{3}\right)\right| + C$

(b) $\log\left|\sin\left(x + \frac{\pi}{3}\right)\right| + \log\left|\sin\left(x - \frac{\pi}{3}\right)\right| + C$

(c) $\log\left|\sin\left(x - \frac{\pi}{3}\right)\right| - \log\left|\sin\left(x + \frac{\pi}{3}\right)\right| + C$

(d) None of these

4. Evaluate : $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

(a) $\log\left|\tan\left(\frac{x}{2}\right)\right| + C$

(b) $-\frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + C$

(c) $\frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + C$

(d) None of these

5. Evaluate : $\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x} dx$

(a) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

(b) $\tan^{-1}(2\tan x) + C$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(d) None of these

6. $\int \frac{dx}{1 - \cos x - \sin x}$ is equal to

(a) $\log\left|1 + \cot\frac{x}{2}\right| + C$ (b) $\log\left|1 - \tan\frac{x}{2}\right| + C$

(c) $\log\left|1 - \cot\frac{x}{2}\right| + C$ (d) $\log\left|1 + \tan\frac{x}{2}\right| + C$

7. Evaluate : $\int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} d\theta$

(a) $-\frac{1}{3}\log|1 + \tan \theta| - \frac{1}{6}\log|\tan^2 \theta - \tan \theta + 1| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan \theta - 1}{\sqrt{3}}\right) + C$

$$(b) -\frac{1}{3} \log |1 + \tan \theta| + \frac{1}{6} \log |\tan^2 \theta - \tan \theta + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan \theta - 1}{\sqrt{3}} \right) + C$$

$$(c) -\frac{1}{3} \log |1 + \tan \theta| + \frac{1}{6} \log |\tan^2 \theta + \tan \theta + 1| - \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan \theta + 1}{\sqrt{3}} \right) + C$$

(d) None of these

8. If $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx = x[f(x) - g(x)] + C$, then

$$(a) f(x) = \log(\log x), g(x) = \frac{1}{\log x}$$

$$(b) f(x) = \log x, g(x) = \frac{1}{\log x}$$

$$(c) f(x) = \frac{1}{\log x}, g(x) = \frac{1}{\log x}$$

$$(d) f(x) = \frac{1}{x \log x}, g(x) = \frac{1}{\log x}$$

9. $\int x \{f(x^2)g''(x^2) - f''(x^2)g(x^2)\} dx$ is equal to

$$(a) f(x^2)g'(x^2) - g(x^2)f'(x^2) + C$$

$$(b) \frac{1}{2} \{f(x^2)g(x^2)f'(x^2)\} + C$$

$$(c) \frac{1}{2} \{f(x^2)g'(x^2) - g(x^2)f'(x^2)\} + C$$

(d) None of these

10. Evaluate : $\int \sqrt{x^2 + 2x + 5} dx$

$$(a) -\frac{1}{2}(x+1)\sqrt{x^2 + 2x + 5} + 2 \log |(x+1) + \sqrt{x^2 + 2x + 5}| + C$$

$$(b) \frac{1}{2}(x+1)\sqrt{x^2 + 2x + 5} + 2 \log |(x+1) + \sqrt{x^2 + 2x + 5}| + C$$

$$(c) -\frac{1}{2}(x+1)\sqrt{x^2 + 2x + 5} - 2 \log |(x+1) + \sqrt{x^2 + 2x + 5}| + C$$

(d) None of these

11. Evaluate : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

$$(a) (\sqrt{1-x})(\sqrt{x}-2) - \sin^{-1} \sqrt{x} + C$$

$$(b) (\sqrt{1-x})(\sqrt{x}+2) + \sin^{-1} \sqrt{x} + C$$

$$(c) (\sqrt{1+x})(\sqrt{x}+2) - \sin^{-1} \sqrt{x} + C$$

$$(d) (\sqrt{1-x})(\sqrt{x}+2) - \sin^{-1} \sqrt{x} + C$$

12. Evaluate : $\int_0^{\pi/4} \sqrt{1 - \sin 2x} dx$

$$(a) \sqrt{2} - 1$$

$$(b) \sqrt{2} + 1$$

$$(c) \sqrt{2}$$

(d) None of these

13. Evaluate : $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$

$$(a) \frac{\pi}{2} - \log 2$$

$$(b) \pi$$

$$(c) \frac{\pi}{4}$$

$$(d) \frac{\pi}{4} - \log 2$$

14. Evaluate : $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$$(a) \frac{\pi}{2}$$

$$(b) \frac{\pi}{4}$$

$$(c) \frac{\pi}{3}$$

(d) None of these

15. $\int_0^{\pi} \sqrt{\frac{\cos 2x + 1}{2}} dx$ is equal to

$$(a) 0$$

$$(b) 2$$

$$(c) -2$$

(d) None of these

16. $\int_{-\pi/4}^{\pi/4} \frac{e^x \cdot \sec^2 x}{e^{2x} - 1} dx$ is equal to

$$(a) 0$$

$$(b) 2$$

$$(c) e$$

(d) None of these

17. AOB is a positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where OA = a, OB = b. The area between the arc AB and chord AB of the ellipse is

$$(a) \pi ab \text{ sq. units}$$

$$(b) (\pi - 2)ab \text{ sq. units}$$

$$(c) \frac{ab(\pi + 2)}{2} \text{ sq. units}$$

$$(d) \frac{ab(\pi - 2)}{4} \text{ sq. units}$$

18. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$.
- (a) $\frac{\pi a^2}{4}$ sq. units (b) $\frac{\pi a^3}{4}$ sq. units
(c) $\frac{\pi a^2}{8}$ sq. units (d) none of these
19. The area of the quadrilateral formed by the tangents at the end points of latus rectum to ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is
- (a) $27/4$ sq. units (b) 9 sq. units
(c) $27/2$ sq. units (d) 27 sq. units
20. Area of the enclosed region between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$ is
- (a) $\left(\frac{8\pi}{3} + 2\sqrt{3}\right)$ sq. units
(b) $\left(\frac{4\pi}{3} + 2\sqrt{3}\right)$ sq. units
(c) $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units
(d) $\left(\frac{4\pi}{3} - 2\sqrt{3}\right)$ sq. units
21. The area bounded by the parabolas $y^2 = 4a(x + a)$ and $y^2 = 4b(x - a)$, $b > a > 0$, is
- (a) $\frac{4\sqrt{2}}{3}b^2\sqrt{\frac{a}{b-a}}$ sq. units
(b) $\frac{8\sqrt{8}}{3}a^2\sqrt{\frac{b}{b-a}}$ sq. units
(c) $\frac{4\sqrt{2}}{3}a^2\sqrt{\frac{b}{b-a}}$ sq. units
(d) $\frac{8\sqrt{8}}{3}b^2\sqrt{\frac{a}{b-a}}$ sq. units
22. The area bounded by $x = -4y^2$ and $x - 1 = -5y^2$ is
- (a) 1 sq. unit (b) $\frac{2}{3}$ sq. units
(c) $\frac{4}{3}$ sq. units (d) 2 sq. units
23. The ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$ is
- (a) 15 : 49 (b) 13 : 37
(c) 15 : 23 (d) 17 : 50
24. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals to
- (a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(b) $\frac{1}{2}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
(d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + C$
25. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to
- (a) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$ (b) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$
(c) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$ (d) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$
26. $\int \frac{1}{x(\log x)\log(\log x)} dx$ is equal to
- (a) $\log(\log x) + C$
(b) $\log|\log(x \log x)| + C$
(c) $\log|\log|\log(\log x)|| + C$
(d) $\log|\log(\log x)| + C$
27. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$ is equal to
- (a) $-\frac{1}{2}\sin 2x + C$ (b) $\frac{1}{2}\sin 2x + C$
(c) $\frac{1}{2}\sin x + C$ (d) $-\frac{1}{2}\sin x + C$
28. The area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \leq 0$, $y \leq x$ and $x \leq \frac{5}{2}$ is
- (a) $\frac{\pi}{6} - \frac{(\sqrt{3}+1)}{8}$ sq. units (b) $\frac{\pi}{6} + \frac{\sqrt{3}+1}{8}$ sq. units
(c) $\frac{\pi}{6} - \frac{(\sqrt{3}-1)}{8}$ sq. units (d) $\frac{\pi}{6} + \frac{(\sqrt{3}-1)}{8}$ sq. units
29. The area (in sq. units) bounded by the parabola $y = \frac{x^2}{4a}$ and the curve $y = \frac{8a^3}{(x^2 + 4a^2)}$ is
- (a) $a^2\left(2\pi + \frac{2}{3}\right)$ (b) $a^2\left(2\pi - \frac{8}{3}\right)$
(c) $a^2\left(\pi + \frac{4}{3}\right)$ (d) $a^2\left(2\pi - \frac{4}{3}\right)$

30. Area bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is (in sq. units)

- (a) $e + \frac{1}{e}$ (b) $e + \frac{1}{e} + 2$
(c) $e + \frac{1}{e} - 2$ (d) $e - \frac{1}{e} + 2$

31. The area (in sq. units) bounded by the curves $y = xe^x$, $y = xe^{-x}$ and the line $x = 1$, is

- (a) $\frac{2}{e}$ (b) $1 + \frac{2}{e}$
(c) $\frac{1}{e}$ (d) $1 + \frac{1}{e}$

32. Evaluate : $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$

- (a) $\sqrt{2}\pi$ (b) $\sqrt{2}\pi$ (c) $\frac{\pi}{2}$ (d) 2π

33. The value of $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ ($n \in \mathbb{N}$) is

- (a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$
(c) π^2 (d) $2\pi^2$

34. If $\int_a^b \frac{x^n}{x^n + (16-x)^n} dx = 6$, then

- (a) $a = 4, b = 12, n \in \mathbb{R}$
(b) $a = 2, b = 14, n \in \mathbb{R}$
(c) $a = -4, b = 20, n \in \mathbb{R}$
(d) $a = 2, b = 8, n \in \mathbb{R}$

Assertion & Reason Based MCQs

For question numbers 35-38, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- (a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false and Statement-II is true.

35. **Statement-I:** $\int \sin 3x \cos 5x dx = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$

Statement-II: $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

36. **Statement-I:** $I = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos^2 x + \sin^4 x} dx$

$$= \frac{\pi}{4} - \frac{1}{4\sqrt{3}} \ln \left(\frac{2-\sqrt{3}}{2+\sqrt{3}} \right)$$

Statement-II: I in statement-I is $\int_0^1 \frac{dx}{1-x^2+x^4}$

37. **Statement-I:** $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{(3+4x^3)dx}{2-\cos\left(|x|+\frac{\pi}{3}\right)} = 4\sqrt{3} \tan^{-1}\left(\frac{1}{2}\right)$

Statement-II: $\int_{-a}^a f(x) dx = \int_0^a (f(x) + f(-x)) dx$

38. **Statement-I:** The area of the smaller region

bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ is $\frac{3}{2}(\pi-2)$ sq. units.

Statement-II: Formula to calculate the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{ab}{4}(\pi-2)$ sq. units.

Matrix-Match

(39-40) : Match Column I with Column II and select the correct answer by choosing an appropriate option.

	Column I	Column II
39.		
(P)	Area under the curve $y = x^2$ between the lines $x = 1$ and $x = 3$ is	(1) $\frac{7}{3}$ sq. units
(Q)	Area enclosed between the x -axis and the curve $y = x^2 - 1$ is	(2) $\frac{26}{3}$ sq. units
(R)	Area enclosed between the x -axis, y -axis and the curve $y^2 = x + 1$ and which lies above the x -axis is	(3) $\frac{4}{3}$ sq. units
(S)	Area bounded by the curve $y^2 = x$, the y -axis and the lines $y = 1$, $y = 2$ is	(4) $\frac{2}{3}$ sq. units

- (a) P-1; Q-2; R-3; S-4 (b) P-2; Q-3; R-1; S-4
(c) P-2; Q-3; R-4; S-1 (d) P-1; Q-3; R-2; S-4

40.

Column I		Column II	
(P)	$\int_0^{\pi/2} \log(\tan x) dx =$	(1)	$\int_0^{\pi/2} \cos^2 x \sin x dx$
(Q)	$\int_0^{\pi/2} \sin^2 x dx =$	(2)	$\int_0^{\pi/2} \log(\cot x) dx$
(R)	$\int_0^{\pi/2} \sin^3 x dx =$	(3)	$\int_0^{\pi/2} \cos^2 x dx$
(S)	$\int_0^{\pi/2} \sin^2 x \cos x dx =$	(4)	$\int_0^{\pi/2} \sin x \sin 2x dx$

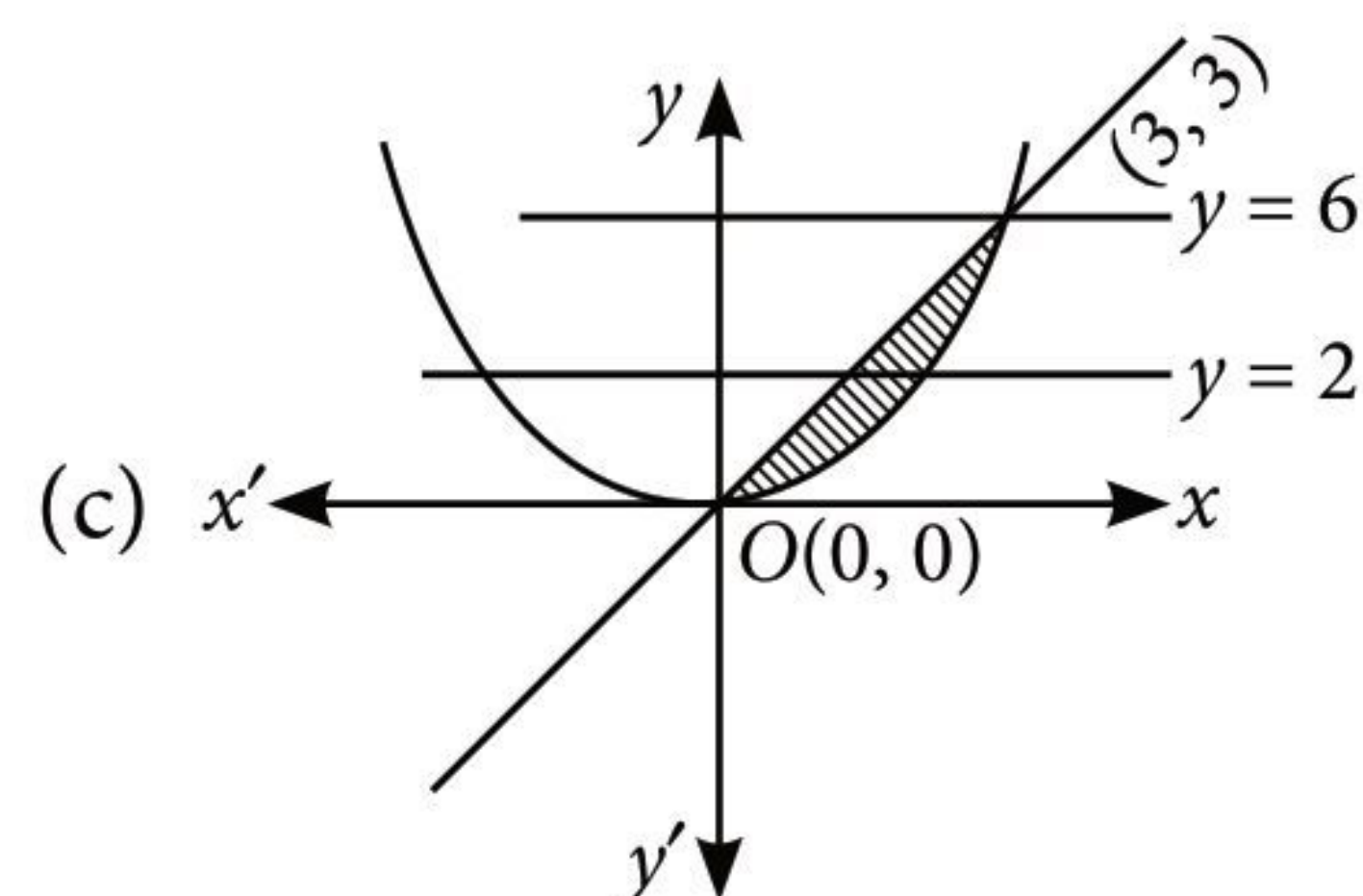
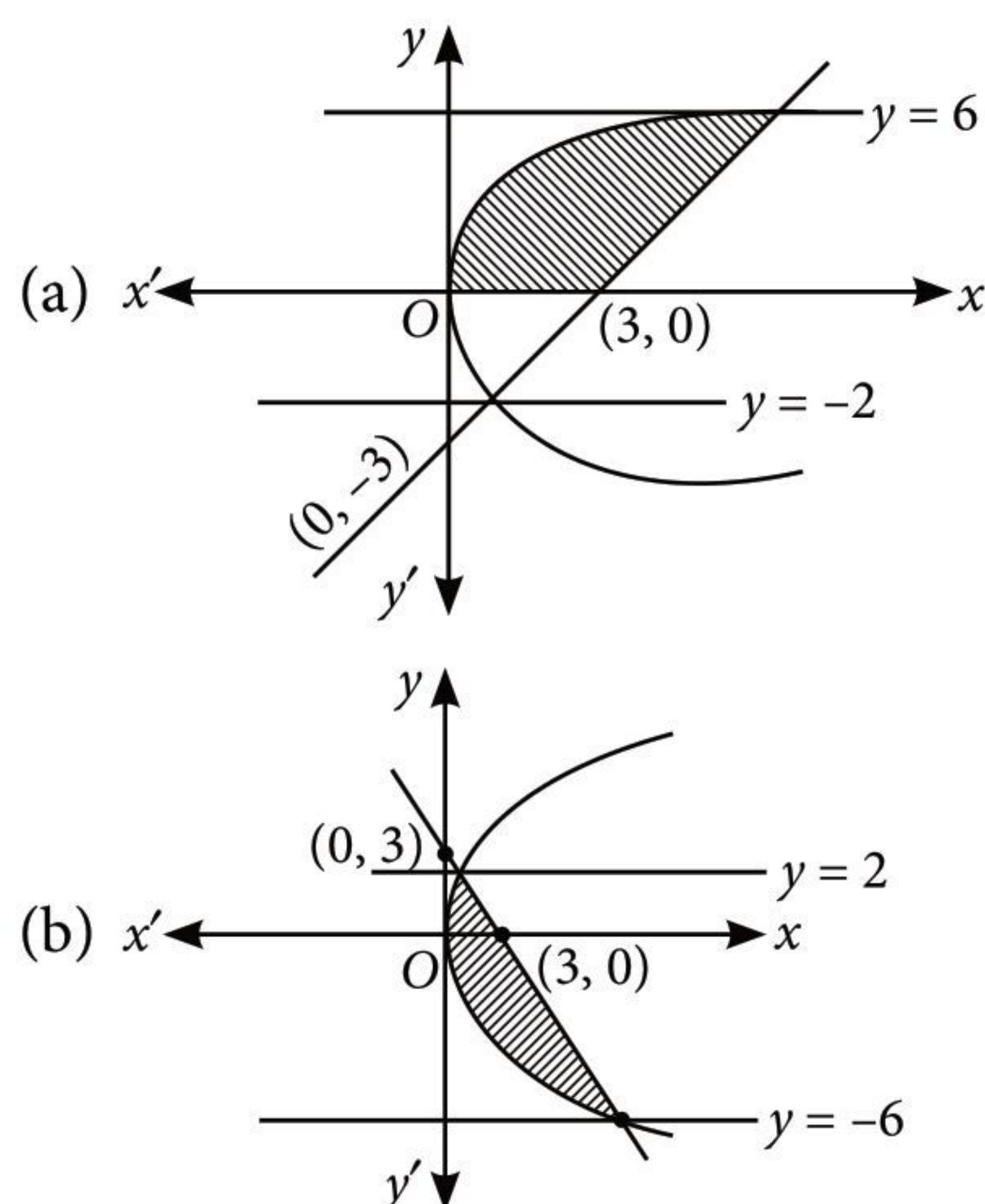
- (a) P-1; Q-3; R-3; S-4 (b) P-1; Q-2; R-3; S-4
(c) P-2; Q-3; R-4; S-1 (d) P-2; Q-1; R-3; S-4

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Consider the following equation of curve $y^2 = 4x$ and straight line $x + y = 3$.

41. The line $x + y = 3$ cuts the x -axis and y -axis respectively at
(a) (0, 2), (2, 0) (b) (3, 3), (0, 0)
(c) (0, 3), (3, 0) (d) (3, 0), (0, 3)
42. Point(s) of intersection of two given equation is (are)
(a) (1, -2), (-9, 6) (b) (2, 1), (-6, 9)
(c) (1, 2), (9, -6) (d) None of these
43. Which of the following shaded portion represent the area bounded by given curves?



(d) None of these

44. Value of the integral $\int_{-6}^2 (3-y) dy$ is
(a) 10 (b) 20 (c) 30 (d) 40
45. Value of area bounded by given curves is
(a) 56 sq. units (b) $\frac{63}{5}$ sq. units
(c) $\frac{64}{3}$ sq. units (d) 31 sq. units

Case II : Read the following passage and answer the questions from 46 to 50.

Ajay is studying in class 12th and today he is going to submit the assignment which was given by his mathematics teacher on the topic of "Integrals".

46. Evaluate : $\int x e^{9x^2} dx$
(a) $\frac{e^{9x^2}}{9} + C$ (b) $\frac{e^{9x^2}}{18} + C$
(c) $\frac{e^{18x^2}}{9} + C$ (d) $\frac{e^{18x^2}}{18} + C$
47. If $\int \frac{1}{x^2 + 64} dx = a \tan^{-1}\left(\frac{x}{b}\right) + C$, then $a + b$ equals
(a) 8 (b) $\frac{64}{9}$ (c) $\frac{65}{8}$ (d) 11
48. If $\int \left(3\sqrt{x} + \frac{4}{x^2} + 6 \right) dx = 64$ at $x = 4$, then the value of C is
(a) 25 (b) 27 (c) 21 (d) 24
49. If $F(t) = \int_0^t (4x^6 + 1) dx$, then find the derivative of $F(t)$.
(a) $24t^5$ (b) $24t^5 + 1$ (c) $6t^4 + 1$ (d) $4t^6 + 1$
50. If $\frac{2x-3}{(x+2)(x-3)}$ is written in partial fraction form as $A\left(\frac{1}{x+2}\right) + B\left(\frac{1}{x-3}\right)$, then $A + B$ equals
(a) 2 (b) -2 (c) 3 (d) -3

SOLUTIONS

1. (a): We have,

$$\begin{aligned} & \int \left\{ (2x-3)^5 + \frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}} + \frac{1}{2-3x} + \sqrt{3x+2} \right\} dx \\ &= \int (2x-3)^5 dx + \int (7x-5)^{-3} dx + \int (5x-4)^{-1/2} dx \\ & \quad + \int \frac{1}{2-3x} dx + \int \sqrt{3x+2} dx \\ &= \frac{(2x-3)^6}{2 \times 6} + \frac{(7x-5)^{-2}}{7 \times (-2)} + \frac{(5x-4)^{1/2}}{5 \times \frac{1}{2}} \\ & \quad + \left(\frac{1}{-3} \right) \log|2-3x| + \frac{(3x+2)^{3/2}}{3 \times \frac{3}{2}} + C \\ &= \frac{1}{12} (2x-3)^6 - \frac{1}{14} (7x-5)^{-2} + \frac{2}{5} \sqrt{5x-4} \\ & \quad - \frac{1}{3} \log|2-3x| + \frac{2}{9} (3x+2)^{3/2} + C \end{aligned}$$

2. (b): Let $I = \int x e^{x^2} dx$

Putting $x^2 = t \Rightarrow 2x dx = dt \Rightarrow x dx = \frac{dt}{2}$, we get

$$I = \frac{1}{2} \int e^t dt = \frac{e^t}{2} + C = \frac{e^{x^2}}{2} + C$$

3. (b): We have, $\int \frac{\sin 2x}{\sin\left(x - \frac{\pi}{3}\right) \sin\left(x + \frac{\pi}{3}\right)} dx$

$$\begin{aligned} &= \int \frac{\sin\left\{\left(x - \frac{\pi}{3}\right) + \left(x + \frac{\pi}{3}\right)\right\}}{\sin\left(x - \frac{\pi}{3}\right) \sin\left(x + \frac{\pi}{3}\right)} dx \\ &= \int \frac{\left\{ \sin\left(x - \frac{\pi}{3}\right) \cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) \sin\left(x + \frac{\pi}{3}\right) \right\}}{\sin\left(x - \frac{\pi}{3}\right) \sin\left(x + \frac{\pi}{3}\right)} dx \\ &= \int \left\{ \cot\left(x + \frac{\pi}{3}\right) + \cot\left(x - \frac{\pi}{3}\right) \right\} dx \\ &= \log \left| \sin\left(x + \frac{\pi}{3}\right) \right| + \log \left| \sin\left(x - \frac{\pi}{3}\right) \right| + C \end{aligned}$$

4. (c): We have, $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

$$= \frac{1}{2} \int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x}$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{1}{\sin\left(x + \frac{\pi}{3}\right)} dx = \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{3}\right) dx \\ &= \frac{1}{2} \log \left| \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) \right| + C \end{aligned}$$

5. (c): Let $I = \int \frac{1}{1 + 3 \sin^2 x + 8 \cos^2 x} dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$\begin{aligned} I &= \int \frac{\sec^2 x}{\sec^2 x + 3 \tan^2 x + 8} dx \\ &= \int \frac{\sec^2 x}{1 + \tan^2 x + 3 \tan^2 x + 8} dx = \int \frac{\sec^2 x dx}{4 \tan^2 x + 9} \end{aligned}$$

Putting $\tan x = t \Rightarrow \sec^2 x dx = dt$, we get

$$\begin{aligned} I &= \int \frac{dt}{4t^2 + 9} = \frac{1}{4} \int \frac{dt}{t^2 + (3/2)^2} = \frac{1}{4} \times \frac{1}{3/2} \tan^{-1}\left(\frac{t}{3/2}\right) + C \\ &= \frac{1}{6} \tan^{-1}\left(\frac{2t}{3}\right) + C = \frac{1}{6} \tan^{-1}\left(\frac{2 \tan x}{3}\right) + C \end{aligned}$$

6. (c): Let $I = \int \frac{dx}{1 - \cos x - \sin x}$

Putting $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ and $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$, we get

$$\begin{aligned} I &= \int \frac{dx}{1 - \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} \\ &= \int \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} = \int \frac{\frac{1}{2} \sec^2 \frac{x}{2} dx}{\tan^2 \frac{x}{2} - \tan \frac{x}{2}} \end{aligned}$$

Putting, $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$, we get

$$\begin{aligned} I &= \int \frac{dt}{t^2 - t} = \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt \\ &= \log(t-1) - \log t + C = \log \left| \frac{t-1}{t} \right| + C \\ &= \log \left| \frac{\tan \frac{x}{2} - 1}{\tan \frac{x}{2}} \right| + C = \log \left| 1 - \cot \frac{x}{2} \right| + C \end{aligned}$$

7. (b): Let $I = \int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} d\theta = \int \frac{\tan \theta \sec^2 \theta}{(1 + \tan^3 \theta)} d\theta$

Putting $\tan \theta = t \Rightarrow \sec^2 \theta d\theta = dt$, we get

$$I = \int \frac{t dt}{(1+t^3)} = \int \frac{t}{(1+t)(t^2-t+1)} dt$$

Let $\frac{t}{(1+t)(1-t+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1-t+t^2}$

$$\Rightarrow t = A(1-t+t^2) + (Bt+C)(t+1) \quad \dots(i)$$

Putting $t = -1$ in (i), we get $A = -\frac{1}{3}$

Comparing the coefficients of t^2 and constant terms on both sides of (i), we get $B = -A = \frac{1}{3}$ and $C = -A = \frac{1}{3}$

$$\begin{aligned} \therefore I &= -\frac{1}{3} \int \frac{1}{1+t} dt + \frac{1}{3} \int \frac{t+1}{t^2-t+1} dt \\ &= -\frac{1}{3} \int \frac{1}{1+t} dt + \frac{1}{6} \int \frac{2t-1+3}{t^2-t+1} dt \\ &= -\frac{1}{3} \int \frac{1}{1+t} dt + \frac{1}{6} \int \frac{2t-1}{t^2-t+1} dt + \frac{1}{2} \int \frac{1}{\left(t-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt \\ &= -\frac{1}{3} \log|1+t| + \frac{1}{6} \log|t^2-t+1| + \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left(\frac{t-\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C \\ &= -\frac{1}{3} \log|1+t| + \frac{1}{6} \log|t^2-t+1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2t-1}{\sqrt{3}} \right) + C \\ &= -\frac{1}{3} \log|1+\tan \theta| + \frac{1}{6} \log|\tan^2 \theta - \tan \theta + 1| \\ &\quad + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan \theta - 1}{\sqrt{3}} \right) + C \end{aligned}$$

8. (a): Given,

$$\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx = x[f(x) - g(x)] + C$$

$$\text{L.H.S.} = \int 1 \cdot \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$$

$$= \log(\log x) \cdot x - \int \left(\frac{1}{(\log x) \times x} \right) dx + \int \frac{1}{(\log x)^2} dx + C$$

$$= x \log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx + C$$

$$= x \log(\log x) - \frac{1}{\log x} \times x + \int \frac{-1}{(\log x)^2} \cdot \frac{x}{x} dx + \int \frac{1}{(\log x)^2} dx + C$$

$$= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx + C$$

$$= x \left[\log(\log x) - \frac{1}{\log x} \right] + C$$

$$\therefore f(x) = \log(\log x), g(x) = \frac{1}{\log x}$$

9. (c): Let $I = \int x\{f(x^2)g''(x^2) - f''(x^2)g(x^2)\} dx$

Putting $x^2 = t \Rightarrow 2x dx = dt$, we get

$$\begin{aligned} I &= \frac{1}{2} \left[\int \{f(t)g''(t)\} dt - \int f''(t)g(t) dt \right] \\ &= \frac{1}{2} [f(t)g'(t) - \int f'(t)g'(t) dt - g(t)f'(t) + \int g'(t)f'(t) dt] + C \\ &= \frac{1}{2} [f(t)g'(t) - g(t)f'(t)] + C \\ &= \frac{1}{2} [f(x^2)g'(x^2) - g(x^2)f'(x^2)] + C \end{aligned}$$

10. (b): We have, $\int \sqrt{x^2 + 2x + 5} dx$

$$= \int \sqrt{(x+1)^2 + (2)^2} dx$$

$$= \frac{1}{2} (x+1) \sqrt{x^2 + 2x + 5} + 2 \log |(x+1) + \sqrt{x^2 + 2x + 5}| + C$$

11. (a): Let, $I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

Putting $x = t^2 \Rightarrow dx = 2t dt$, we get

$$\begin{aligned} I &= 2 \int \sqrt{\frac{1-t}{1+t}} \cdot t dt = 2 \int \frac{t(1-t)}{\sqrt{1-t^2}} dt \\ &= \int \frac{2t}{\sqrt{1-t^2}} dt + 2 \int \frac{-t^2}{\sqrt{1-t^2}} dt \\ &= \int \frac{2t}{\sqrt{1-t^2}} dt + 2 \int \frac{(1-t^2)-1}{\sqrt{1-t^2}} dt \\ &= -\int \frac{-2t}{\sqrt{1-t^2}} dt + 2 \int \sqrt{1-t^2} dt - 2 \int \frac{1}{\sqrt{1-t^2}} dt \\ &= -2\sqrt{1-t^2} + 2 \times \frac{1}{2} [t\sqrt{1-t^2} + \sin^{-1} t] - 2 \sin^{-1} t + C \\ &= -2\sqrt{1-t^2} + t\sqrt{1-t^2} - \sin^{-1} t + C \\ &= \sqrt{(1-x)}(\sqrt{x}-2) - \sin^{-1} \sqrt{x} + C \end{aligned}$$

12. (a): Let $I = \int_0^{\pi/4} \sqrt{1 - \sin 2x} dx$

$$\begin{aligned}
&= \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x} dx \\
&= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2} dx = \int_0^{\pi/4} |(\cos x - \sin x)| dx \\
&= \int_0^{\pi/4} (\cos x - \sin x) dx \quad [\because 0 < x < \pi/4, \cos x > \sin x] \\
&= [\sin x + \cos x]_0^{\pi/4} = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1
\end{aligned}$$

13. (a) : We have, $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = \int_0^1 2 \tan^{-1} x dx$

$$\begin{aligned}
&= 2 \int_0^1 \tan^{-1} x \cdot 1 dx = 2 \left[x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{1}{1+x^2} \cdot x dx \\
&= 2 \left[x \tan^{-1} x \right]_0^1 - \frac{1}{2} [\log(1+x^2)]_0^1 \\
&= 2 \left[(1 \cdot \tan^{-1} 1 - 0 \tan^{-1} 0) - \frac{1}{2} (\log 2 - \log 1) \right] \\
&= 2 \left[\left(\frac{\pi}{4} - 0 \right) - \frac{1}{2} \log 2 \right] = \frac{\pi}{2} - \log 2
\end{aligned}$$

14. (a) : Let $I = \int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$$= \int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + (1 - \sin^2 x)^2} dx$$

Let $\sin^2 x = t \Rightarrow 2 \sin x \cos x dx = dt \Rightarrow \sin 2x dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore I = \int_0^1 \frac{dt}{t^2 + (1-t)^2} = \frac{1}{2} \int_0^1 \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= [\tan^{-1}(2t-1)]_0^1 = \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{2}$$

15. (b) : $\int_0^{\pi} \sqrt{\frac{\cos 2x + 1}{2}} dx = \int_0^{\pi} \sqrt{\cos^2 x} dx = \int_0^{\pi} |\cos x| dx$

$$= \int_0^{\pi/2} \cos x dx - \int_{\pi/2}^{\pi} \cos x dx = [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$$

$$= \left[\sin \frac{\pi}{2} - \sin 0 \right] - \left[\sin \pi - \sin \frac{\pi}{2} \right] = 2$$

16. (a) : Let $I = \int_{-\pi/4}^{\pi/4} \frac{e^x \cdot \sec^2 x}{e^{2x} - 1} dx$

Here, $f(x) = \frac{e^x \sec^2 x}{e^{2x} - 1}$

$$\therefore f(-x) = \frac{e^{-x} \sec^2(-x)}{e^{-2x} - 1} = \frac{e^x \sec^2 x}{1 - e^{2x}} = -\frac{e^x \sec^2 x}{e^{2x} - 1} = -f(x)$$

$\therefore I = 0$ ($\because f(x)$ is an odd function)

17. (d) : Required area

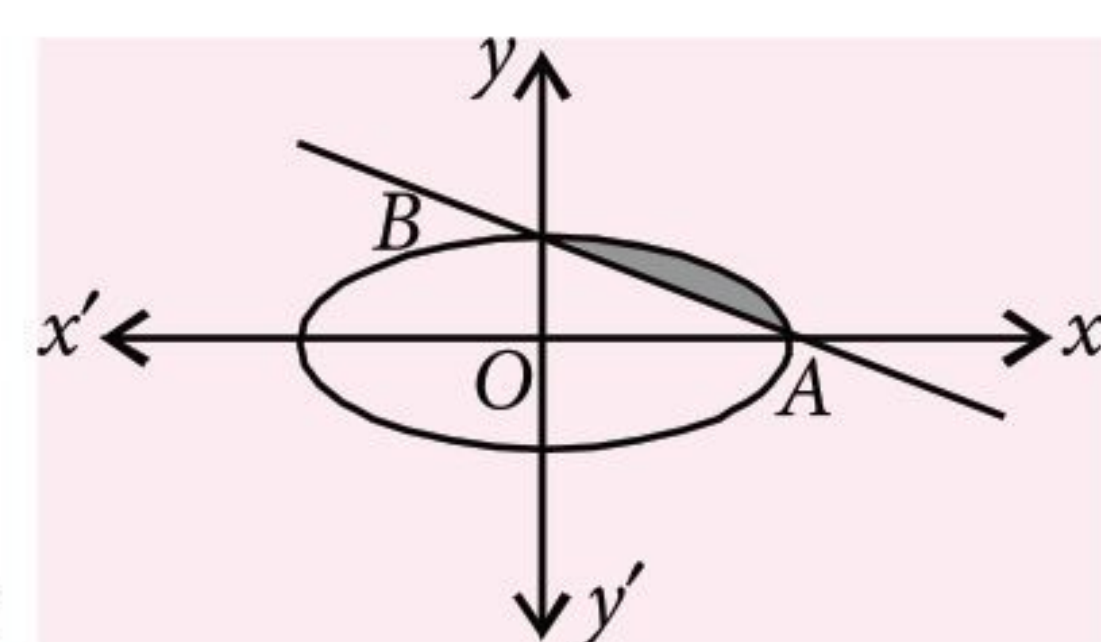
$$= \int_0^a \left(\frac{b}{a} \sqrt{a^2 - x^2} - \frac{b}{a} (a - x) \right) dx$$

$$= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - \frac{b}{a} \int_0^a (a - x) dx$$

$$= \frac{b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a - \frac{b}{a} \left[ax - \frac{x^2}{2} \right]_0^a$$

$$= \frac{b}{a} \left[\frac{a^2}{2} \sin^{-1} 1 \right] - \frac{b}{a} \left[a^2 - \frac{a^2}{2} \right]$$

$$= \frac{ab}{2} \frac{\pi}{2} - ba \left(\frac{1}{2} \right) = \frac{ab}{4} (\pi - 2) \text{ sq. units}$$



18. (a) : We have given the equation of curve

$$y = \sqrt{a^2 - x^2}$$

$$\Rightarrow y^2 = a^2 - x^2$$

$$\Rightarrow y^2 + x^2 = a^2, \text{ a circle with}$$

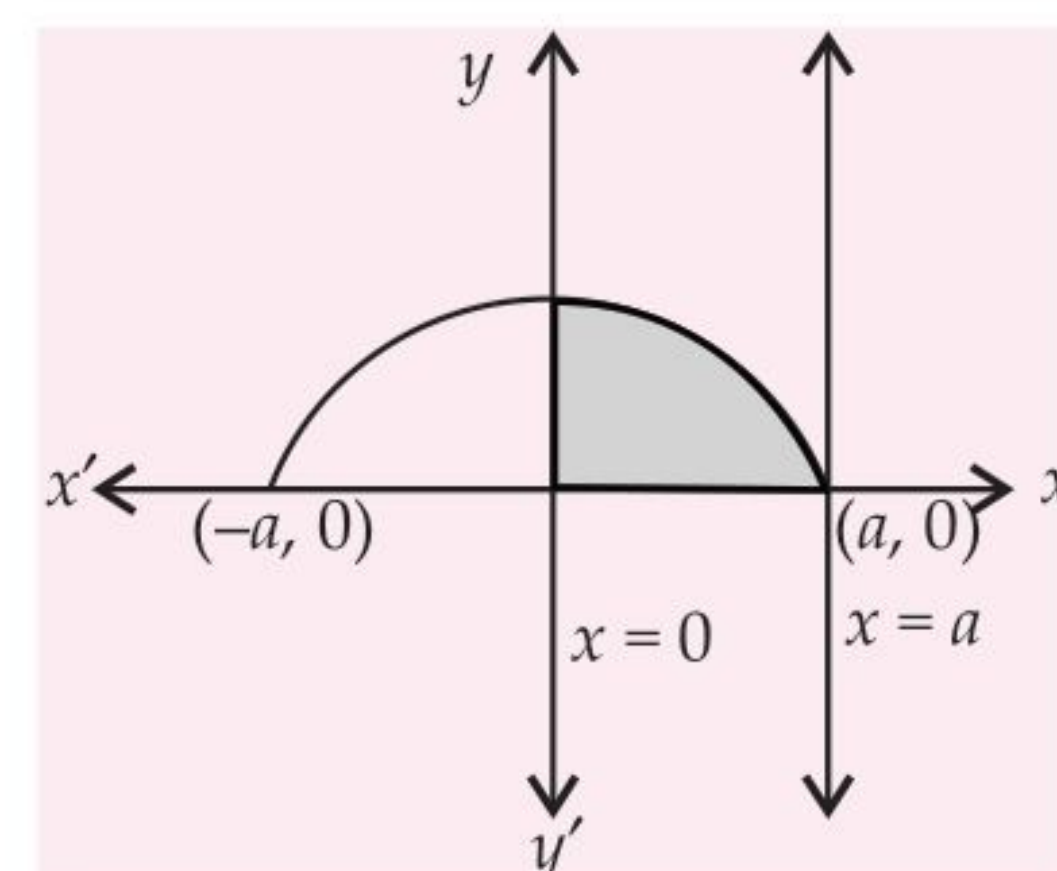
centre (0, 0) and radius a

Thus the required area

= area of the shaded region

$$= \int_0^a \sqrt{a^2 - x^2} dx$$

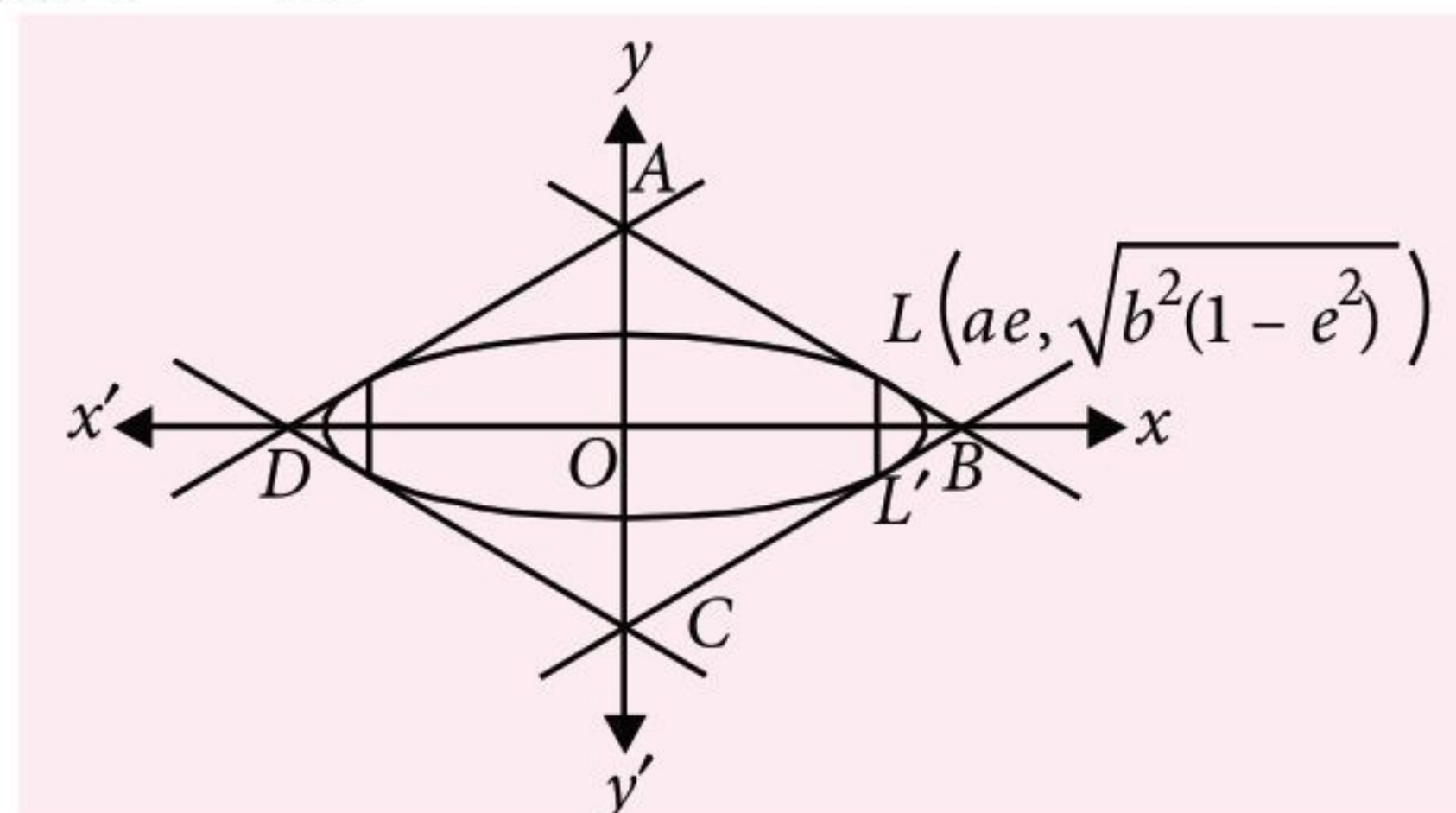
$$= \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a = \frac{\pi a^2}{4} \text{ sq. units}$$



19. (d) : Given equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$.

On comparing it with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a = 3 \text{ and } b = \sqrt{5}$$



To find tangents at the end points of latus rectum let us

$$\text{first find } ae, \text{ i.e., } ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$$

By symmetry the quadrilateral is rhombus.

So, area of rhombus is four times the area of the right angled triangle formed by the tangent and axes in the first quadrant.

$$\Rightarrow \text{Equation of tangent at } [ae, \sqrt{b^2(1-e^2)}] = \left(2, \frac{5}{3}\right) \text{ is}$$

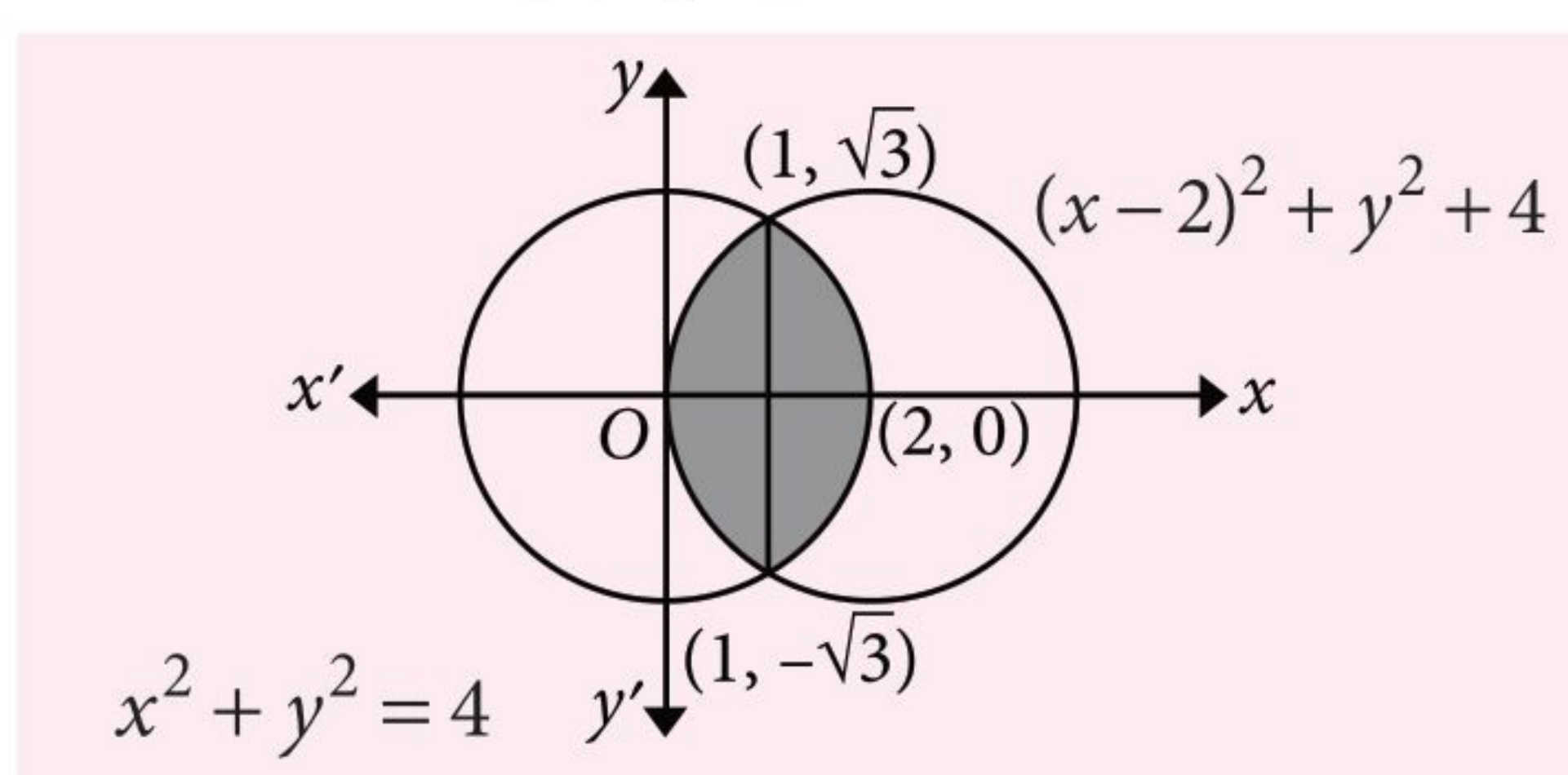
$$\frac{2}{9}x + \frac{5}{3} \cdot \frac{y}{5} = 1 \Rightarrow 2x + 3y = 9$$

Now, area of quadrilateral ABCD = 4(area of $\triangle AOB$)

$$= 4 \int_0^{9/2} \frac{9-2x}{3} dx = \left| \frac{4}{3} [9x - x^2]_0^{9/2} \right|$$

$$= \frac{4}{3} \left[\frac{81}{2} - \frac{81}{4} \right] = 27 \text{ sq. units}$$

20. (c) : We have, $x^2 + y^2 = 4$... (i)
a circle with centre (0, 0) and radius 2



and $(x-2)^2 + y^2 = 4$... (ii)

a circle with centre (2, 0) and radius 2.

Points of intersection of (i) and (ii) are $x = 1$ and $y = \pm \sqrt{3}$

Required area = area of shaded region

$$= 2 \left[\int_0^1 \sqrt{4-(x-2)^2} dx + \int_1^2 \sqrt{4-x^2} dx \right]$$

$$= 2 \left[\frac{1}{2} (x-2) \sqrt{4-(x-2)^2} + \frac{1}{2} \times 4 \sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^1$$

$$+ 2 \left[\frac{1}{2} x \sqrt{4-x^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{x}{2} \right]_1^2$$

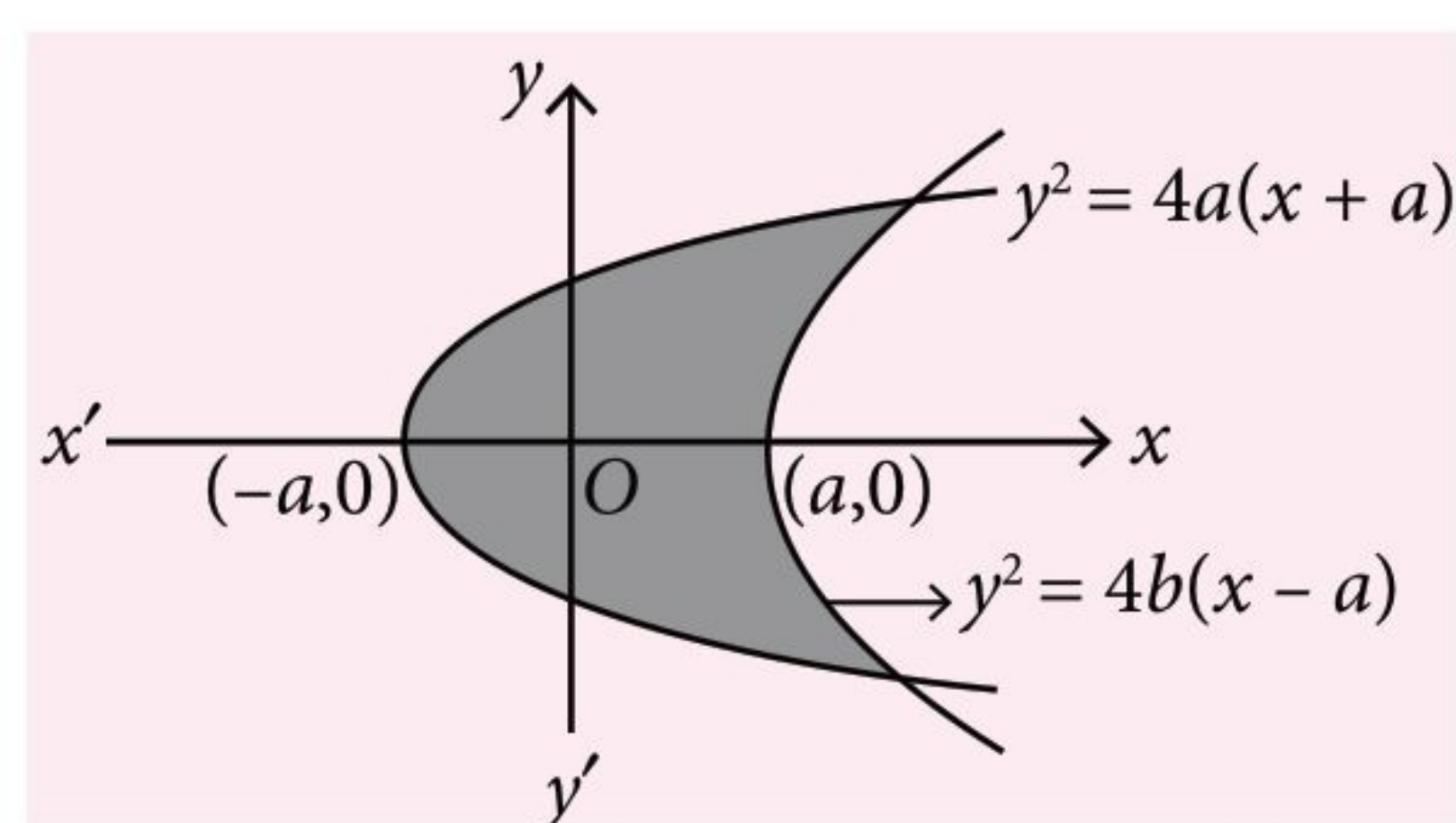
$$= \left[\left(-\sqrt{3} - 4 \times \frac{\pi}{6} \right) + 4 \times \frac{\pi}{2} \right] + \left[4 \times \frac{\pi}{2} - \sqrt{3} - 4 \times \frac{\pi}{6} \right]$$

$$= \left(\frac{8\pi}{3} - 2\sqrt{3} \right) \text{ sq. units}$$

21. (b) : We have, $y^2 = 4a(x+a)$... (i), a parabola with vertex $(-a, 0)$

and $y^2 = 4b(x-a)$... (ii), a parabola with vertex $(a, 0)$

Solving (i) and (ii), we get $y = \pm a \sqrt{\frac{8b}{b-a}}$



$$\text{Required area} = 2 \int_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}} \left(\left(\frac{y^2}{4b} + a \right) - \left(\frac{y^2}{4a} - a \right) \right) dy$$

$$= 2 \int_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}} \left(2a - \frac{(b-a)y^2}{4ab} \right) dy = 2 \left[2ay - \frac{b-a}{12ab} y^3 \right]_0^{\frac{a\sqrt{8b}}{\sqrt{b-a}}}$$

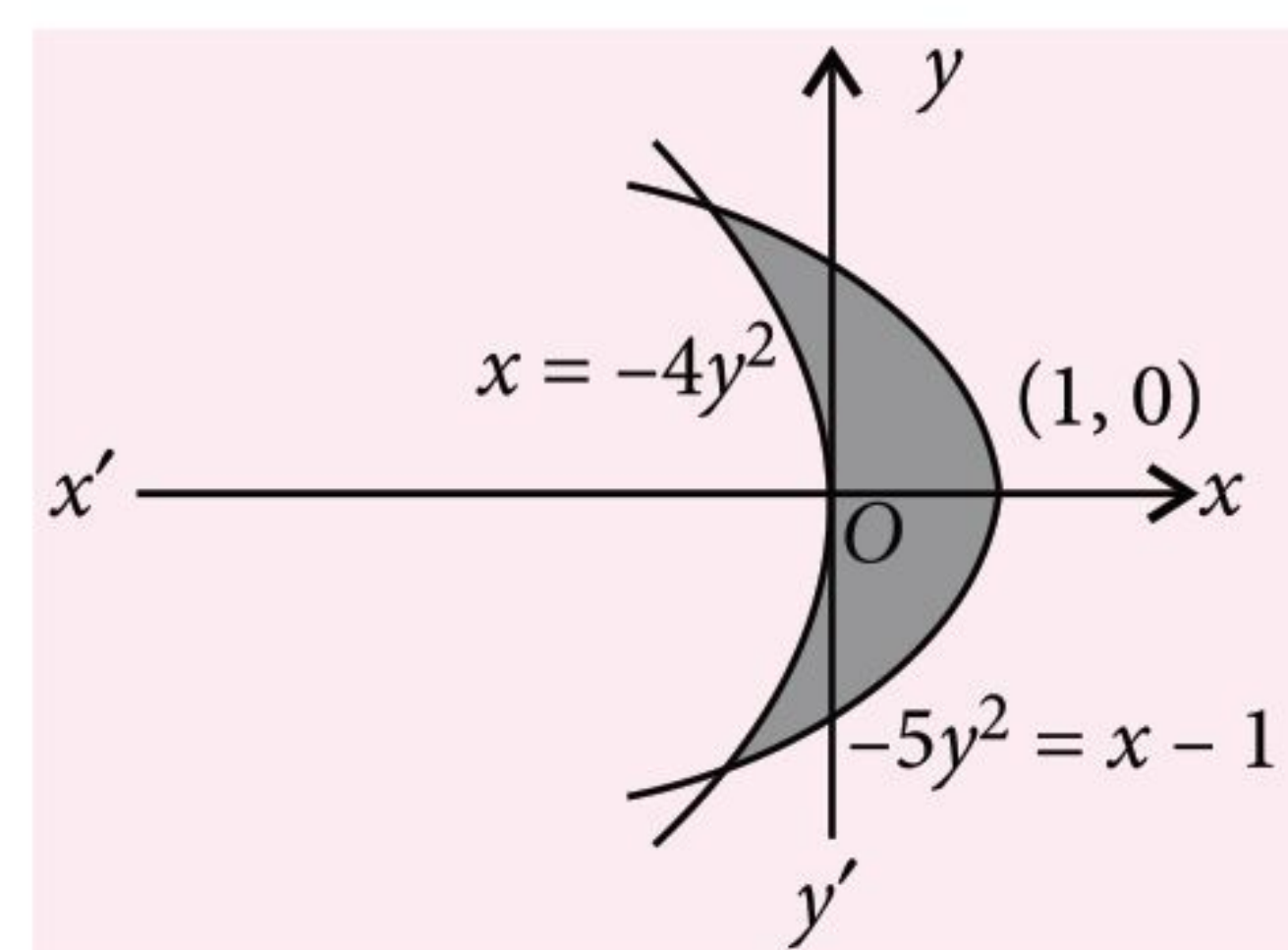
$$= 2 \left[2a \times \frac{a\sqrt{8b}}{\sqrt{b-a}} - \frac{b-a}{12ab} \left(\frac{a\sqrt{8b}}{\sqrt{b-a}} \right)^3 \right]$$

$$= 4a^2 \sqrt{\frac{8b}{b-a}} - \frac{4}{3} a^2 \sqrt{\frac{8b}{b-a}} = \frac{8a^2}{3} \sqrt{\frac{8b}{b-a}} \text{ sq. units}$$

22. (c) : We have $x = -4y^2$... (i), a parabola with vertex (0, 0),

and $-5y^2 = x - 1$... (ii), a parabola with vertex (1, 0).

Solving (i) and (ii), we get $y = \pm 1$

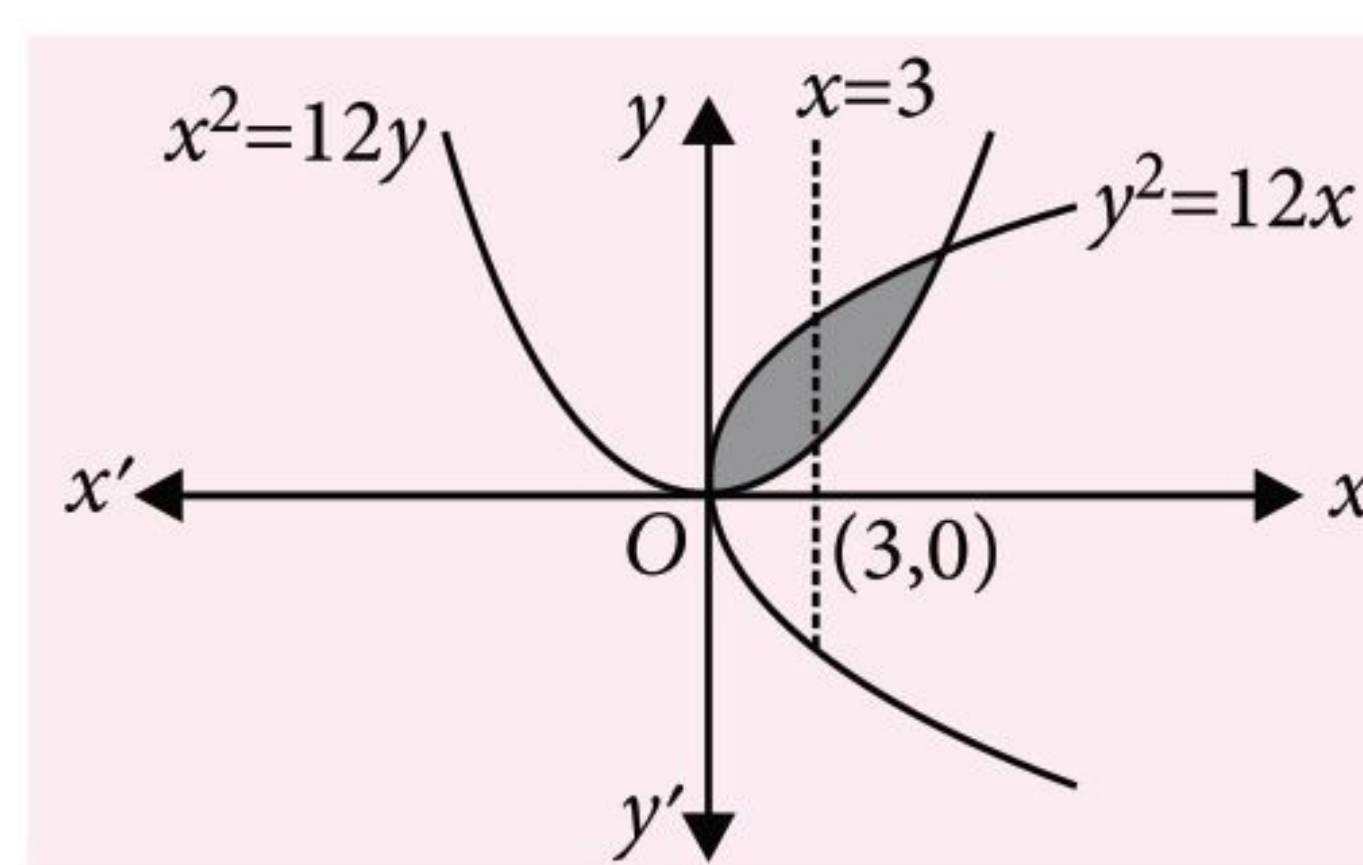


$$\text{Required area} = \int_{-1}^1 (1 - 5y^2 - (-4y^2)) dy$$

$$= \int_{-1}^1 (1 - y^2) dy$$

$$= 2 \int_0^1 (1 - y^2) dy = 2 \left[y - \frac{y^3}{3} \right]_0^1 = 2 \left(1 - \frac{1}{3} \right) = \frac{4}{3} \text{ sq. units}$$

23. (a) : We have $y^2 = 12x$ and $x^2 = 12y$, parabolas with vertex (0, 0).



The curves intersect at $x = 0, 12$.

$$\text{Required ratio} = \frac{\int_0^3 \left(\sqrt{12x} - \frac{x^2}{12} \right) dx}{\int_3^{12} \left(\sqrt{12x} - \frac{x^2}{12} \right) dx} = \frac{\left[\sqrt{12} \cdot \frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{36} \right]_0^3}{\left[\sqrt{12} \cdot \frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{36} \right]_3^{12}} = \frac{15}{49}$$

24. (d) : Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sqrt{2} \left(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1 \right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos \left(x + \frac{\pi}{4} \right)} = \frac{1}{\sqrt{2}} \int \frac{dx}{2 \sin^2 \left(\frac{x}{2} + \frac{\pi}{8} \right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2 \left(\frac{x}{2} + \frac{\pi}{8} \right) dx$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{-\cot \left(\frac{x}{2} + \frac{\pi}{8} \right)}{\frac{1}{2}} + C = \frac{-1}{\sqrt{2}} \cot \left(\frac{x}{2} + \frac{\pi}{8} \right) + C$$

25. (b) : Let $I = \int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

$$= \int \frac{\frac{2x^{12} + 5x^9}{x^{15}}}{\left(\frac{x^5 + x^3 + 1}{x^5} \right)^3} dx = \int \frac{\frac{2}{x^3} + \frac{5}{x^6}}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^3} dx$$

Put $1 + \frac{1}{x^2} + \frac{1}{x^5} = u$, so that $\frac{du}{dx} = -\left(\frac{2}{x^3} + \frac{5}{x^6} \right)$

$$\therefore I = -\int \frac{du}{u^3} = \frac{1}{2u^2} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

26. (d) : Let $I = \int \frac{1}{x(\log x) \log(\log x)} dx$

Let $\log(\log x) = t \Rightarrow \frac{1}{x \log x} dx = dt$

$$\therefore I = \int \frac{dt}{t} = \log |t| + C = \log |\log(\log x)| + C$$

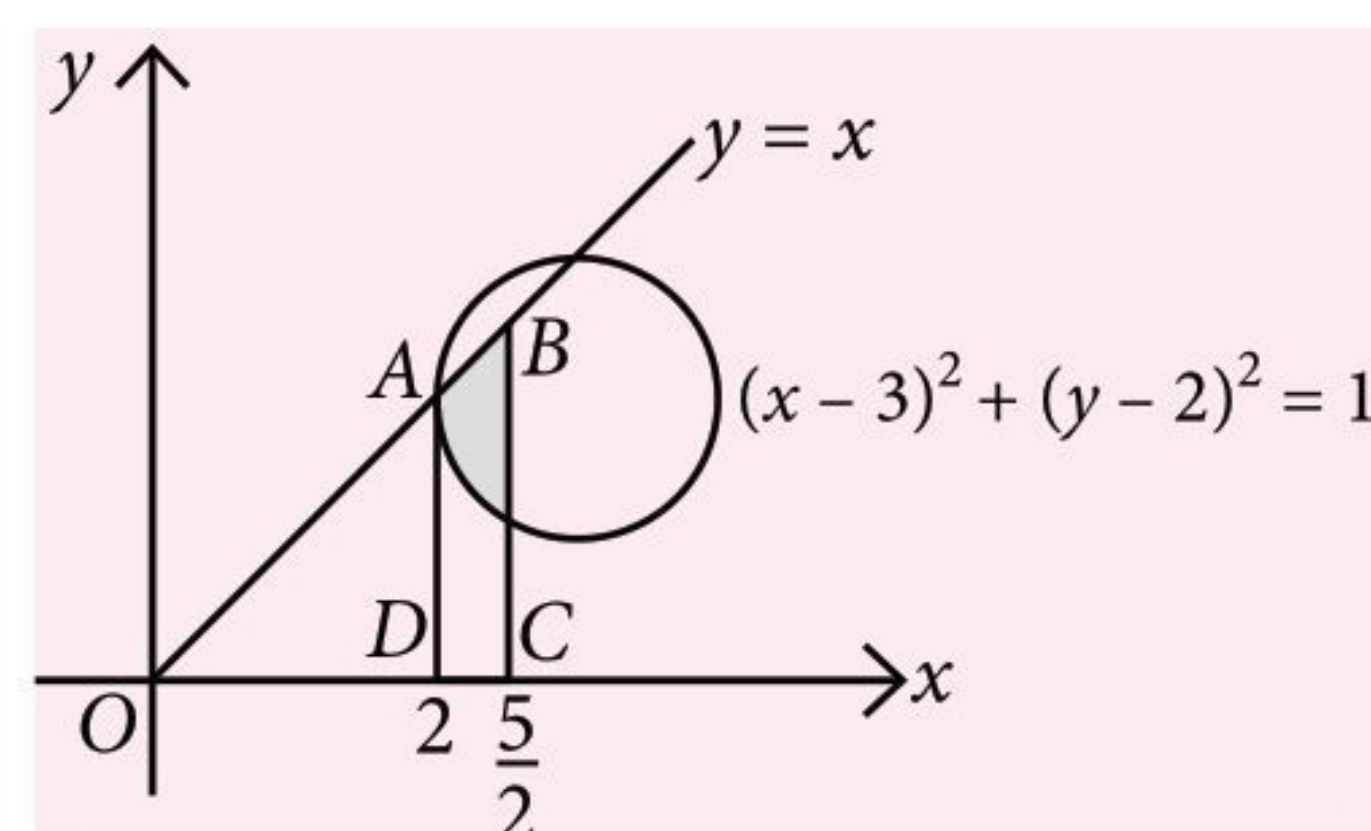
27. (a) : Let $I = \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x) dx}{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x}$

$$= \int (\sin^4 x - \cos^4 x) dx$$

$$= \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) dx$$

$$= -\int \cos 2x dx = -\frac{1}{2} \sin 2x + C$$

28. (a) : The circle is $(x-3)^2 + (y-2)^2 = 1$



$$\therefore \text{Required area} = \left| \int_2^{5/2} x dx - \int_2^{5/2} \left\{ 2 + \sqrt{1 - (x-3)^2} \right\} dx \right|$$

$$= \left| \left[\frac{x^2}{2} \right]_2^{5/2} - [2x]_2^{5/2} - \left[\frac{x-3}{2} \sqrt{1 - (x-3)^2} + \frac{1}{2} \sin^{-1}(x-3) \right]_2^{5/2} \right|$$

$$= \left| \frac{1}{2} \left[\frac{25}{4} - 4 \right] - 2 \left[\frac{5}{2} - 2 \right] - \left[\frac{\frac{5}{2} - 3}{2} \sqrt{1 - \left(\frac{5}{2} - 3 \right)^2} + \frac{1}{2} \sin^{-1} \left(\frac{5}{2} - 3 \right) \right] \right.$$

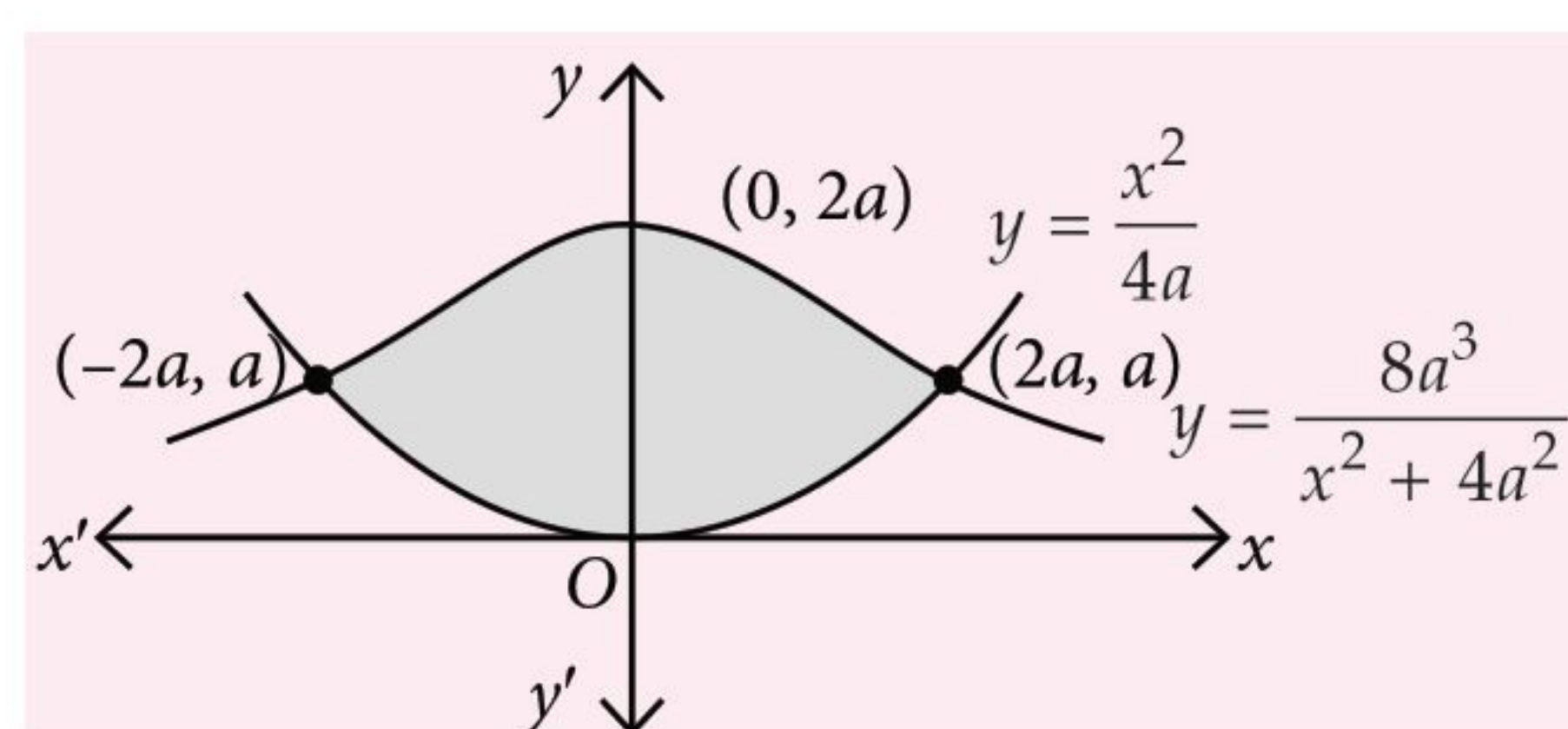
$$\left. + \frac{1}{2} \sin^{-1} \left(\frac{5}{2} - 3 \right) - \frac{2-3}{2} \sqrt{1 - (2-3)^2} - \frac{1}{2} \sin^{-1}(2-3) \right|$$

$$= \left| \frac{9}{8} - 1 - \frac{1}{2} \left(-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right) \right| = \left| \frac{1}{8} - \frac{\pi}{6} + \frac{\sqrt{3}}{8} \right|$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8} - \frac{1}{8} \text{ sq. units}$$

29. (d) : Area bounded by the parabola $y = \frac{x^2}{4a}$ and the curve $y = \frac{8a^3}{(x^2 + 4a^2)}$ is shaded.

$$\therefore \text{Required area} = 2 \left[\int_0^{2a} \frac{8a^3}{x^2 + 4a^2} dx - \int_0^{2a} \frac{x^2}{4a} dx \right]$$



$$= 2 \left[\frac{8a^3}{2a} \tan^{-1} \frac{x}{2a} - \frac{1}{4a} \frac{x^3}{3} \right]_0^{2a}$$

$$= 2 \left[4a^2 \frac{\pi}{4} - \frac{1}{12a} 8a^3 \right] = a^2 \left(2\pi - \frac{4}{3} \right) \text{ sq. units}$$

30. (c) : Let $y_1 = e^x$ and $y_2 = e^{-x}$.

Required area = Area of shaded region in the figure

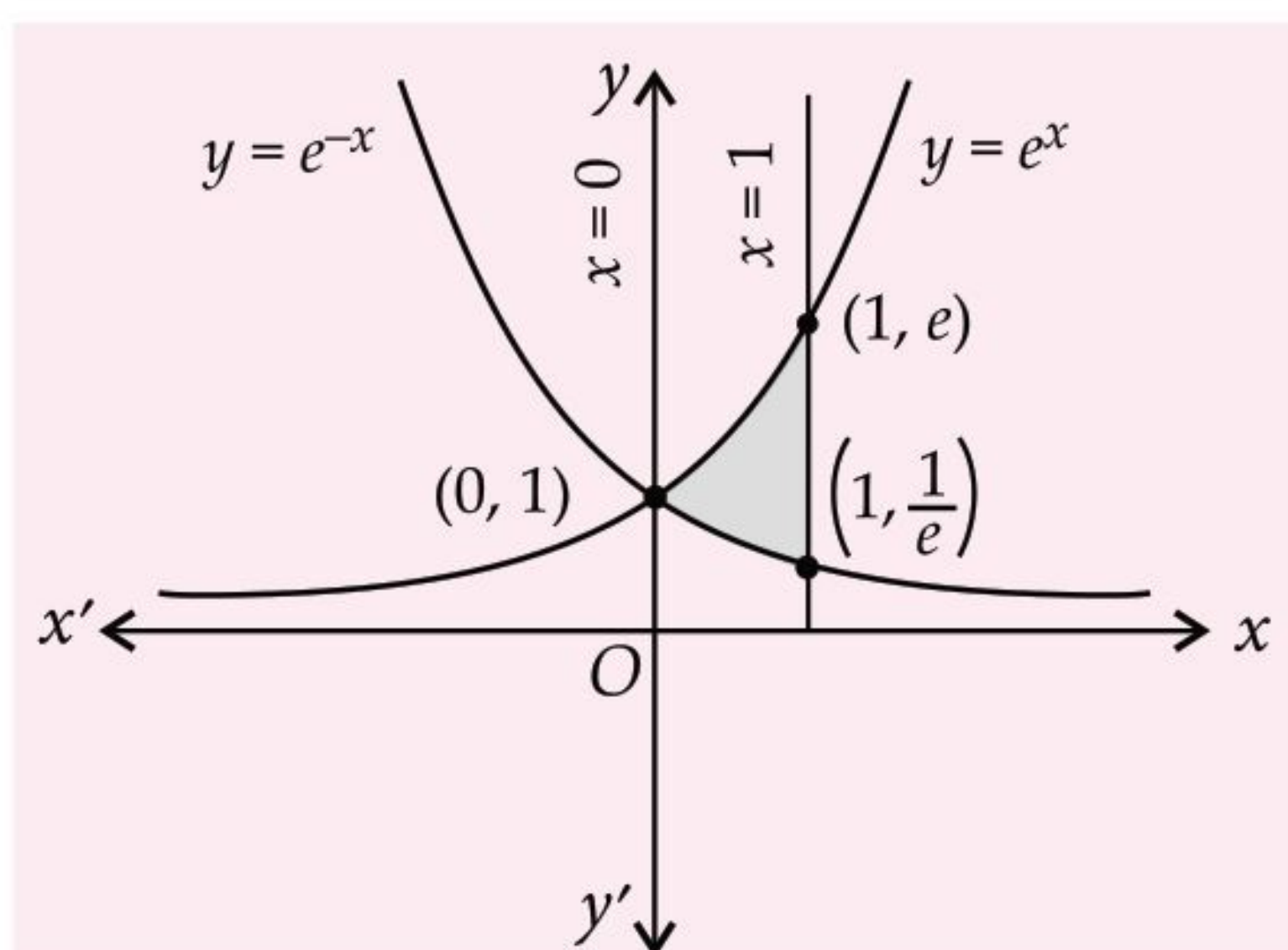
$$= \int_0^1 (y_1 - y_2) dx$$

$$= \int_0^1 (e^x - e^{-x}) dx$$

$$= \left[e^x + e^{-x} \right]_0^1$$

$$= e + e^{-1} - e^0 - e^0$$

$$= (e + e^{-1} - 2) \text{ sq. units}$$



31. (a) : Area of shaded region

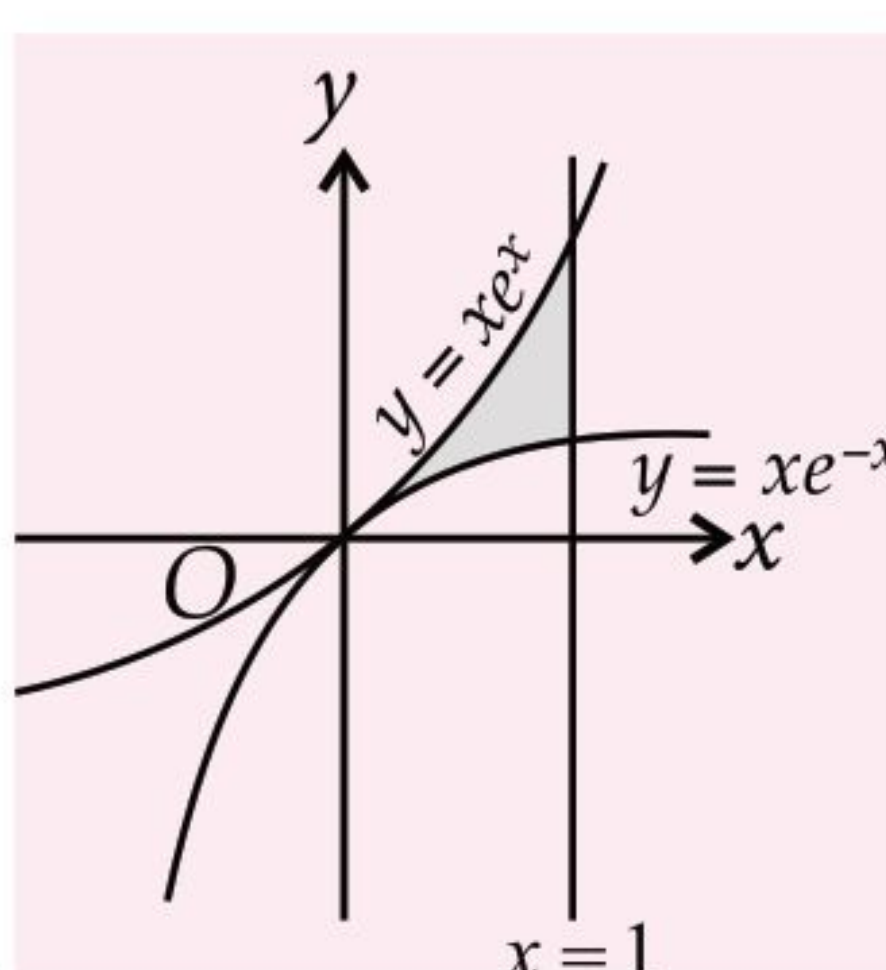
$$= \int_0^1 (xe^x - xe^{-x}) dx$$

$$= \int_0^1 x(e^x - e^{-x}) dx$$

$$= \left[\left[x(e^x + e^{-x}) \right]_0^1 - \int_0^1 (e^x + e^{-x}) dx \right]$$

$$= \left[x(e^x + e^{-x}) - (e^x - e^{-x}) \right]_0^1$$

$$= [e + e^{-1} - e + e^{-1} - (+1 - 1)] = \frac{2}{e} \text{ sq. units}$$



32. (a) : Let $I = \int_0^{\pi/2} \{ \sqrt{\tan x} + \sqrt{\cot x} \} dx$

$$= \int_0^{\pi/2} \left\{ \sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}} \right\} dx = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$= \sqrt{2} \int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx = \sqrt{2} \int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

Let $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$

Now $x = 0 \Rightarrow t = -1$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\pi/2} \{ \sqrt{\tan x} + \sqrt{\cot x} \} dx$$

$$= \sqrt{2} \int_{-1}^1 \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} [\sin^{-1} t]_{-1}^1 = 2\sqrt{2} \left(\frac{\pi}{2} \right) = \sqrt{2} \pi$$

33. (c) : Let $I = \int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$... (i)

$$\Rightarrow I = \int_0^{2\pi} \frac{(2\pi - x) \sin^{2n}(2\pi - x)}{\sin^{2n}(2\pi - x) + \cos^{2n}(2\pi - x)} dx \text{ (by property)}$$

$$\Rightarrow I = \int_0^{2\pi} \frac{(2\pi - x) \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx \quad \dots \text{(ii)}$$

Adding (i) and (ii), we get

$$2I = 2\pi \int_0^{2\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$$

$$\therefore I = \pi \int_0^{2\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx = \pi \cdot 2 \int_0^{\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$$

(by property)

$$\therefore I = 4\pi \int_0^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx \text{ (by property)} \quad \dots \text{(iii)}$$

$$= 4\pi \int_0^{\pi/2} \frac{\sin^{2n} \left(\frac{\pi}{2} - x \right)}{\sin^{2n} \left(\frac{\pi}{2} - x \right) + \cos^{2n} \left(\frac{\pi}{2} - x \right)} dx \text{ (by property)}$$

$$\Rightarrow I = 4\pi \int_0^{\pi/2} \frac{\cos^{2n} x}{\cos^{2n} x + \sin^{2n} x} dx \quad \dots \text{(iv)}$$

Adding (iii) and (iv), we get

$$2I = 4\pi \int_0^{\pi/2} 1 \cdot dx = 4\pi \cdot \frac{\pi}{2} \quad \therefore I = \pi^2$$

34. (b) : $\int_a^b \frac{x^n}{x^n + (16-x)^n} dx = 6$... (i)

Let $a + b = 16$, then by property

$$\int_a^b \frac{(16-x)^n}{(16-x)^n + x^n} dx = 6 \quad \dots \text{(ii)}$$

Adding (i) and (ii), we get

$$\int_a^b 1 \cdot dx = 12 \Rightarrow b - a = 12$$

Solving $a + b = 16$ and $b - a = 12$, we get $a = 2$, $b = 14$ and $n \in R$



BUY ONLINE

Now you can buy

MTG Books & Magazines

Log on to : **www.mtg.in**

35. (a) : We have, $\int \sin 3x \cos 5x \, dx$

$$\begin{aligned} &= \frac{1}{2} \int 2 \cos 5x \sin 3x \, dx \\ &= \frac{1}{2} \int (\sin 8x - \sin 2x) \, dx = \frac{1}{2} \left[\int \sin 8x \, dx - \int \sin 2x \, dx \right] \\ &= \frac{1}{2} \left[\frac{-\cos 8x}{8} \right] - \frac{1}{2} \left[\frac{-\cos 2x}{2} \right] + C \\ &= \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C \end{aligned}$$

$\therefore$ Both Statement-I and statement-II are true and Statement-II is the correct explanation of Statement-I.

36. (a) : $I = \int_0^{\frac{\pi}{4}} \frac{\cos x \, dx}{\cos^2 x + \sin^4 x} + \int_0^{\frac{\pi}{4}} \frac{\sin x \, dx}{\cos^2 x + \sin^4 x}$

$$= \int_0^{\frac{\pi}{4}} \frac{\cos x \, dx}{1 - \sin^2 x + \sin^4 x} + \int_0^{\frac{\pi}{4}} \frac{\sin x \, dx}{\cos^2 x + (1 - \cos^2 x)^2}$$

Putting $\sin x = t$ in the first integral and $\cos x = t$ in the second integral, we get

$$I = \int_0^{\frac{1}{\sqrt{2}}} \frac{dt}{1 - t^2 + t^4} + \int_{\frac{1}{\sqrt{2}}}^1 \frac{dt}{1 - t^2 + t^4} = \int_0^1 \frac{dt}{1 - t^2 + t^4}$$

$$= \frac{1}{2} \int_0^1 \frac{\frac{2}{t^2} dt}{t^2 + \frac{1}{t^2} - 1} = \frac{1}{2} \int_0^1 \frac{\left(1 + \frac{1}{t^2}\right) - \left(1 - \frac{1}{t^2}\right)}{t^2 + \frac{1}{t^2} - 1} dt$$

$$= \frac{1}{2} \int_0^1 \frac{d\left(t - \frac{1}{t}\right)}{\left(t - \frac{1}{t}\right)^2 + 1} - \frac{1}{2} \int_0^1 \frac{d\left(t + \frac{1}{t}\right)}{\left(t + \frac{1}{t}\right)^2 - 3}$$

$$= \frac{1}{2} \left[\tan^{-1} \left(t - \frac{1}{t} \right) \right]_0^1 - \frac{1}{4\sqrt{3}} \left[\ln \left| \frac{t + \frac{1}{t} - \sqrt{3}}{t + \frac{1}{t} + \sqrt{3}} \right| \right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{4\sqrt{3}} \ln \left(\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \right)$$

37. (a) : $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{(3 + 4x^3)}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx = \int_0^{\frac{\pi}{3}} \frac{6 \, dx}{2 - \cos\left(x + \frac{\pi}{3}\right)}$

Now, $x + \frac{\pi}{3} = t \Rightarrow I = 6 \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dt}{2 - \cos t}$

Now, $\tan \frac{t}{2} = u \Rightarrow I = 12 \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{du}{1 + 3u^2}$

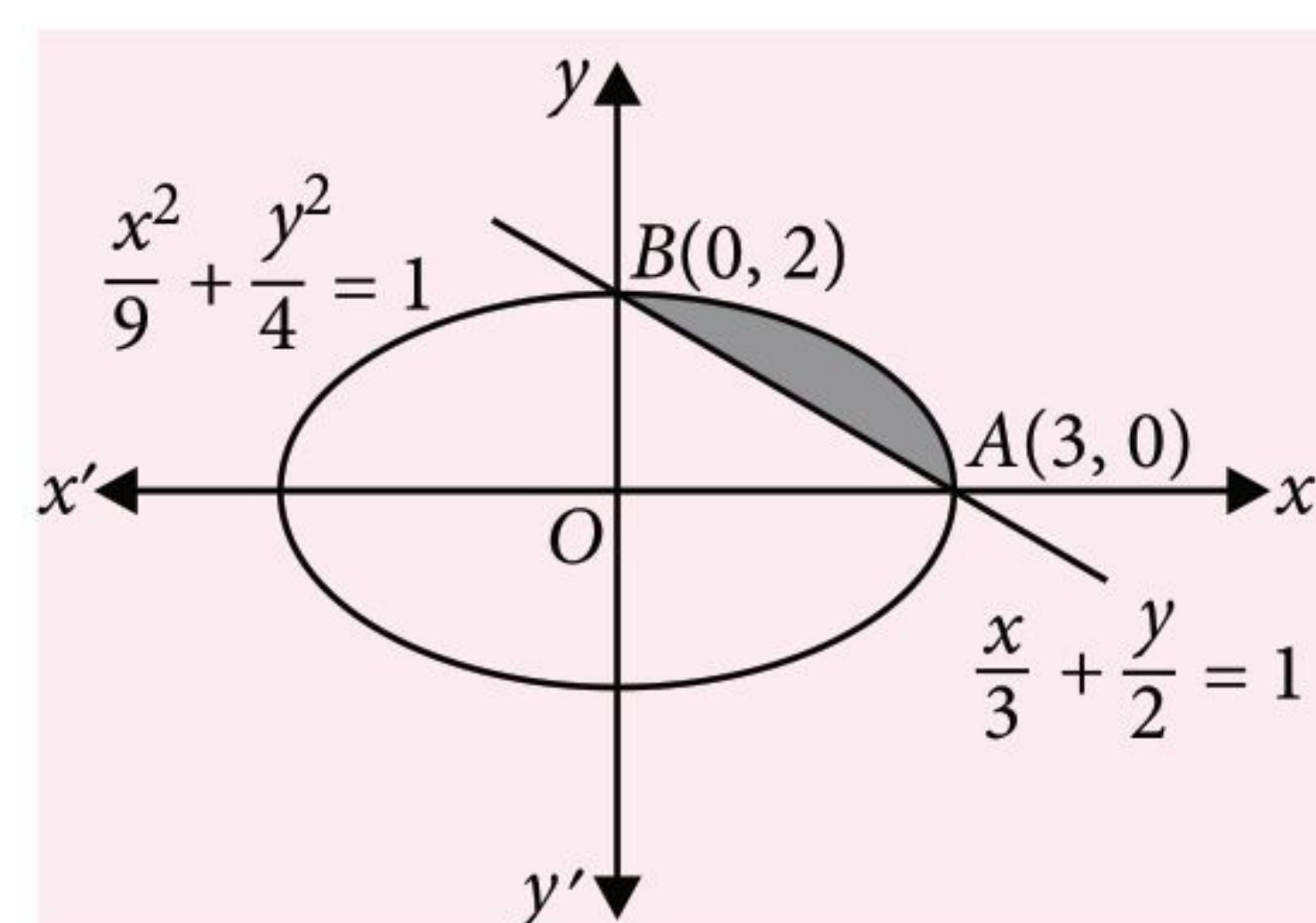
$$= 4\sqrt{3} \left[\tan^{-1} \sqrt{3}u \right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} = 4\sqrt{3} (\tan^{-1} 3 - \tan^{-1} 1)$$

$$= 4\sqrt{3} \tan^{-1} \left(\frac{3-1}{1+3} \right) = 4\sqrt{3} \tan^{-1} \left(\frac{1}{2} \right)$$

38. (a) : Given curve is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, which represents an ellipse with centre at $(0, 0)$

and equation of line is $\frac{x}{3} + \frac{y}{2} = 1$

Clearly, intersection points are $A(3, 0)$ and $B(0, 2)$.



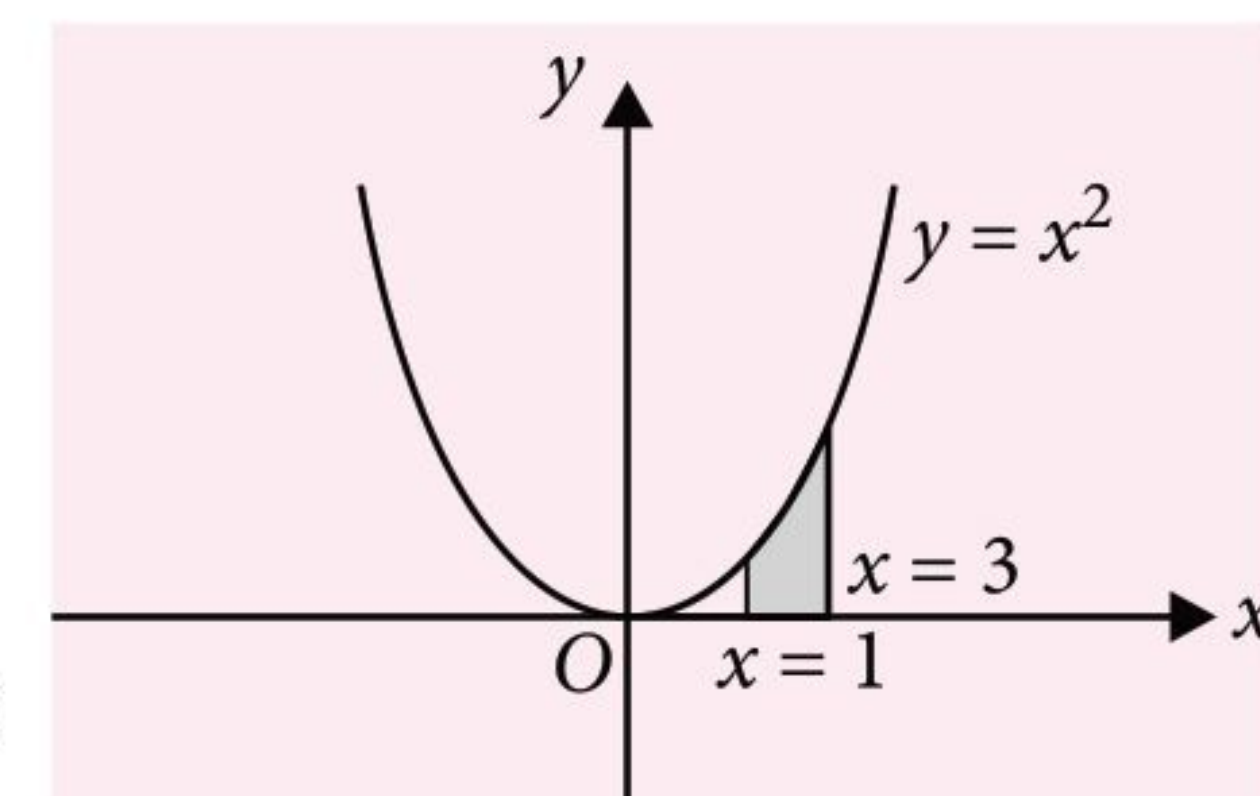
$\therefore$ Required area = area of shaded region

$$\begin{aligned} &= \int_0^3 2\sqrt{1 - \frac{x^2}{9}} \, dx - \int_0^3 2\left(1 - \frac{x}{3}\right) dx \\ &= \frac{2}{3} \int_0^3 \sqrt{3^2 - x^2} \, dx - \frac{2}{3} \int_0^3 (3 - x) \, dx \\ &= \frac{2}{3} \left[\frac{x}{2} \sqrt{3^2 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[3x - \frac{x^2}{2} \right]_0^3 \\ &= \frac{2}{3} \left[0 + \frac{9}{2} \sin^{-1}(1) - 0 \right] - \frac{2}{3} \left[9 - \frac{9}{2} - 0 \right] \\ &= 3 \left(\frac{\pi}{2} \right) - (3) = 3 \left(\frac{\pi}{2} - 1 \right) = \frac{3}{2} (\pi - 2) \text{ sq. units} \end{aligned}$$

39. (c) : (P) Required area

$$= \int_1^3 x^2 \, dx = \left[\frac{x^3}{3} \right]_1^3$$

$$= \frac{1}{3} (27 - 1) = \frac{26}{3} \text{ sq. units}$$



(Q) The equation

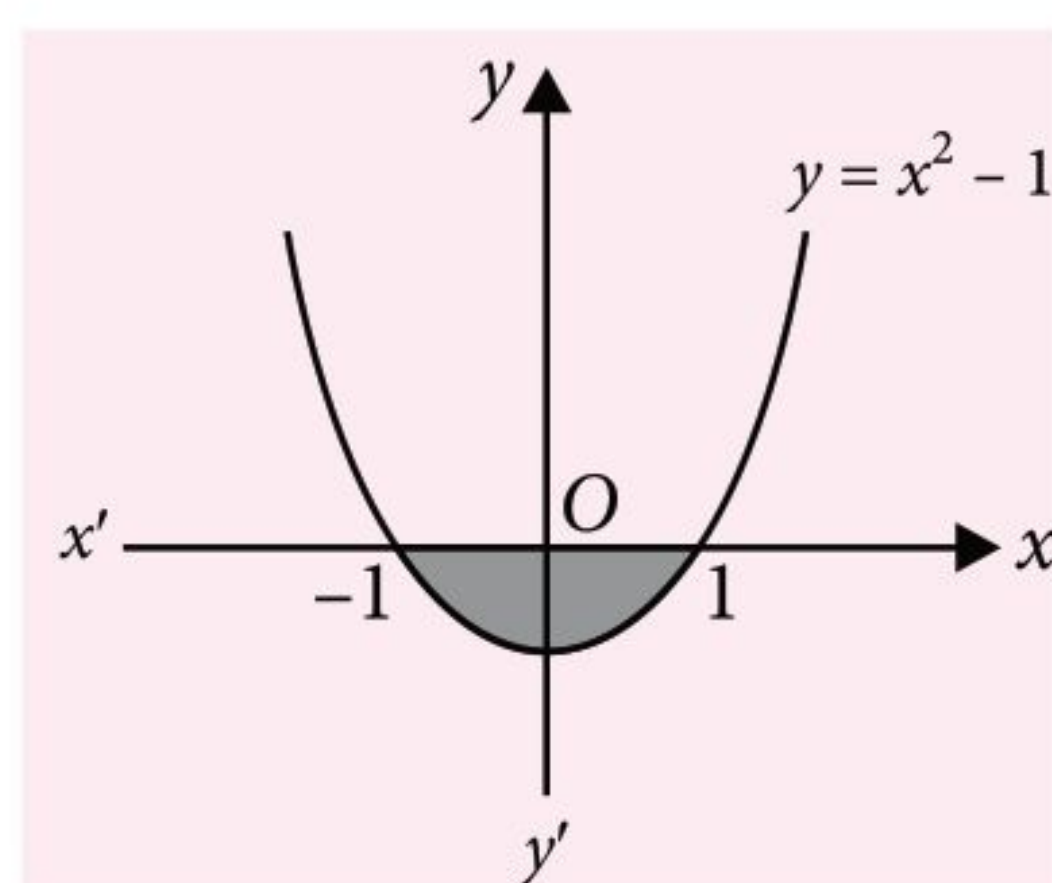
$y = x^2 - 1$ represents a parabola with vertex at $(0, -1)$.

It cuts x -axis where $y = 0$

$$\therefore x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$\therefore$ Required area

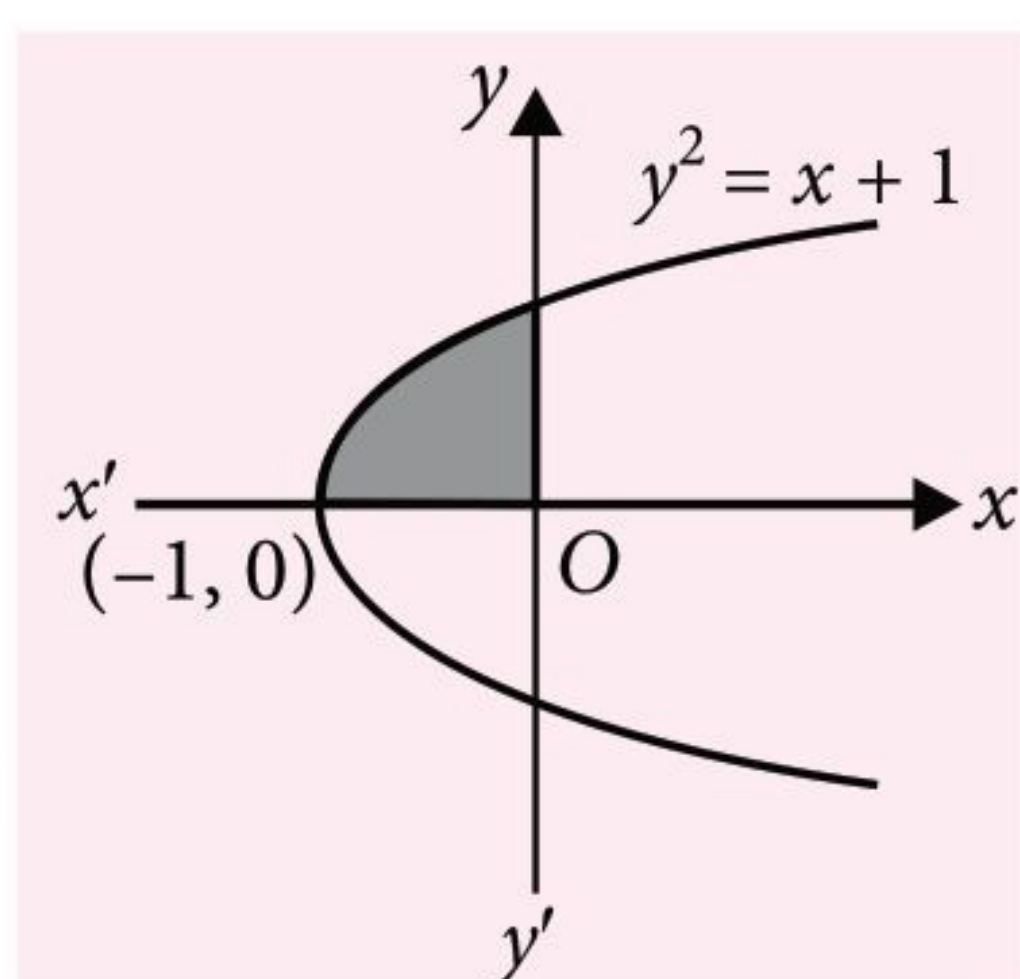
$$\begin{aligned} &= \left| \int_{-1}^1 (x^2 - 1) dx \right| = \left| \left[\frac{x^3}{3} - x \right]_{-1}^1 \right| \\ &= \left| \frac{1}{3}(1+1) - (1+1) \right| = \left| \frac{2}{3} - 2 \right| = \frac{4}{3} \text{ sq. units} \end{aligned}$$



(R) The equation $y^2 = x + 1$ represents a parabola with vertex at $(-1, 0)$.

$\therefore$ Required area

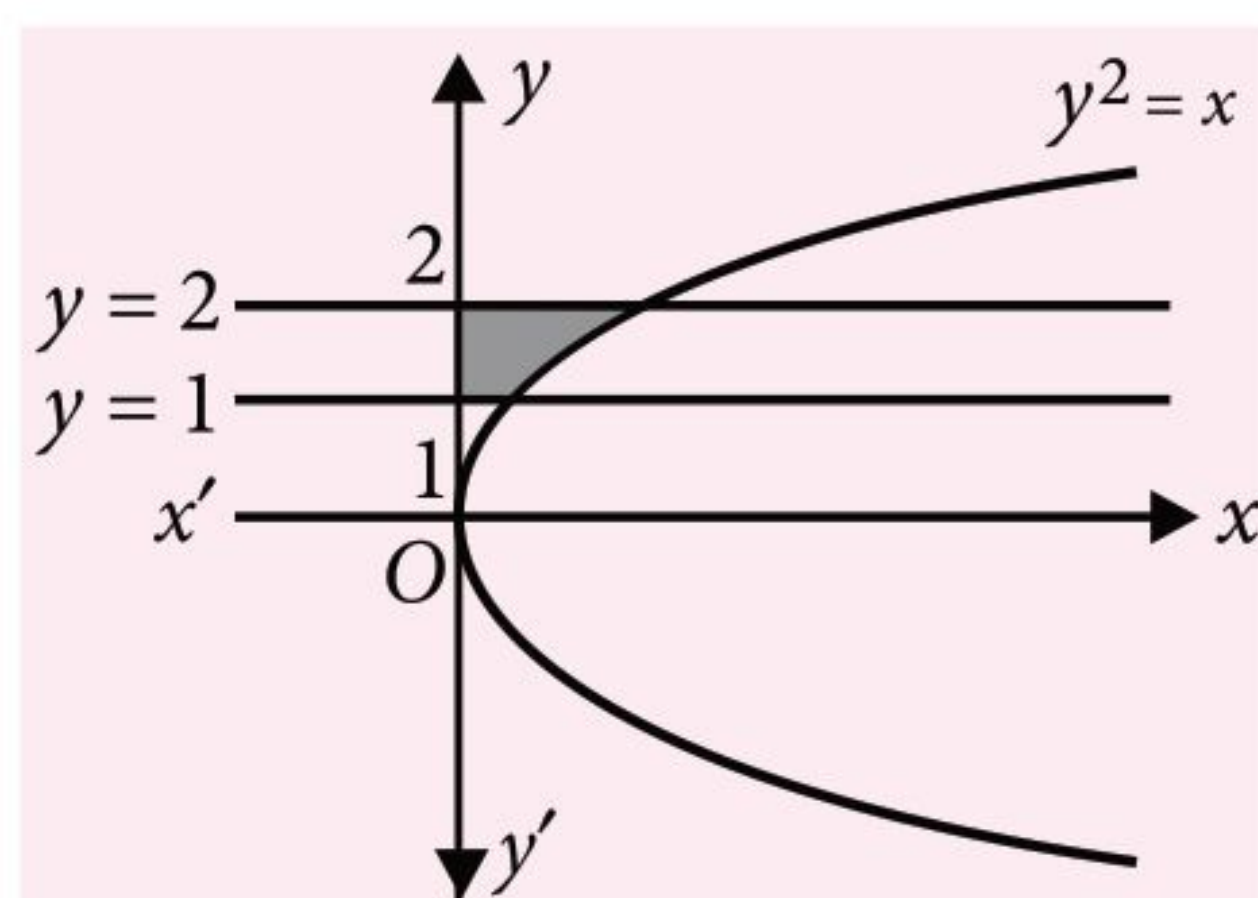
$$\begin{aligned} &= \int_{-1}^0 \sqrt{x+1} dx = \frac{2}{3} \left[(x+1)^{3/2} \right]_{-1}^0 \\ &= \frac{2}{3} [(1)^{3/2} - 0] = \frac{2}{3} \text{ sq. units} \end{aligned}$$



(S) The equation $y^2 = x$ represents a parabola with vertex at $(0, 0)$.

$\therefore$ Required area

$$\begin{aligned} &= \int_1^2 y^2 dy \\ &= \frac{1}{3} [y^3]_1^2 = \frac{1}{3} (8 - 1) = \frac{7}{3} \text{ sq. units} \end{aligned}$$



40. (c) : (P) $\int_0^{\pi/2} \log(\tan x) dx$

$$= \int_0^{\pi/2} \log \left\{ \tan \left(\frac{\pi}{2} - x \right) \right\} dx = \int_0^{\pi/2} \log(\cot) dx$$

(Q) $\int_0^{\pi/2} \sin^2 x dx = \int_0^{\pi/2} \left[\sin \left(\frac{\pi}{2} - x \right) \right]^2 dx = \int_0^{\pi/2} \cos^2 x dx$

(R) $I = \int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \left(\frac{3}{4} \sin x - \frac{1}{4} \sin 3x \right) dx$

$$= \left[-\frac{3}{4} \cos x \right]_0^{\pi/2} + \left[\frac{1}{4 \cdot 3} \cos 3x \right]_0^{\pi/2}$$

$$= -\frac{3}{4} (0 - 1) + \frac{1}{12} \left(\cos \frac{3\pi}{2} - \cos 0 \right) = \frac{3}{4} - \frac{1}{12} = \frac{2}{3}$$

Also, $\int_0^{\pi/2} \frac{2 \sin x \sin 2x}{2} dx = \frac{1}{2} \int_0^{\pi/2} (\cos x - \cos 3x) dx$

$$= \frac{1}{2} \left[\sin x \right]_0^{\pi/2} - \left[\frac{\sin 3x}{3} \right]_0^{\pi/2} = \frac{1}{2} \left[(1 - 0) - \frac{1}{3} (-1 - 0) \right]$$

$$= \frac{1}{2} \left[1 + \frac{1}{3} \right] = \frac{2}{3}$$

$$\therefore \int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \sin x \sin 2x dx$$

(S) $\int_0^{\pi/2} \sin^2 x \cos x dx = \int_0^{\pi/2} (1 - \cos^2 x) \cos x dx$

$$= \int_0^{\pi/2} \cos x dx - \int_0^{\pi/2} \cos^3 x dx$$

$$= \int_0^{\pi/2} \cos \left(\frac{\pi}{2} - x \right) dx - \int_0^{\pi/2} \left\{ \cos \left(\frac{\pi}{2} - x \right) \right\}^3 dx$$

$$= \int_0^{\pi/2} \sin x dx - \int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} (\sin x - \sin^3 x) dx$$

$$= \int_0^{\pi/2} (1 - \sin^2 x) \sin x dx = \int_0^{\pi/2} \cos^2 x \sin x dx$$

41. (d) : Line $x + y = 3$ cuts the x -axis and y -axis at $(3, 0)$ and $(0, 3)$ respectively.

[Since, at x -axis, $y = 0$ and at y -axis, $x = 0$]

42. (c) : We have, $y^2 = 4x$... (i) and $x + y = 3$... (ii)

From (i) and (ii), we get $y^2 = 4(3 - y)$

$$\Rightarrow y^2 + 4y - 12 = 0 \Rightarrow y^2 + 6y - 2y - 12 = 0$$

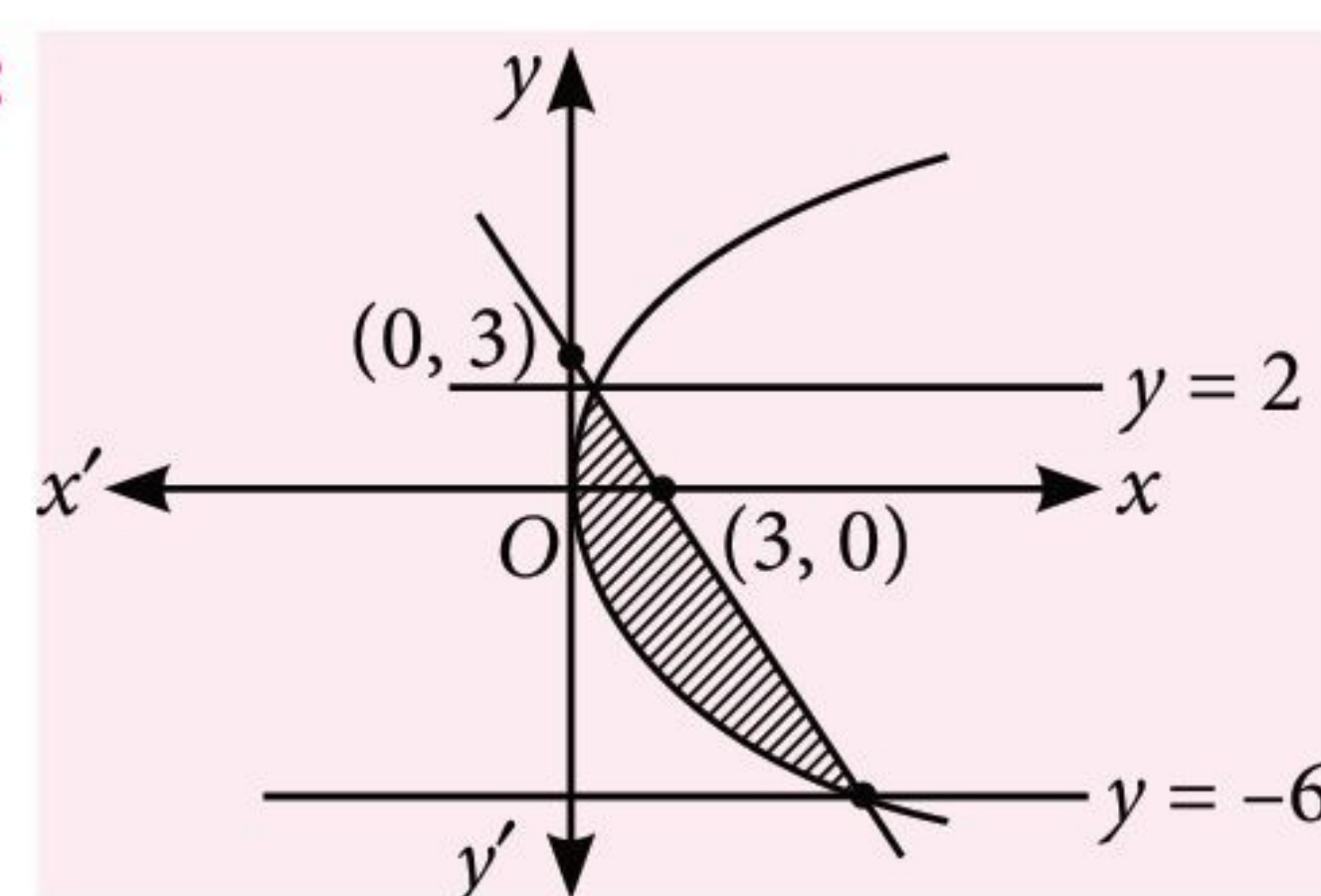
$$\Rightarrow y(y + 6) - 2(y + 6) = 0 \Rightarrow (y + 6)(y - 2) = 0$$

$$\Rightarrow y = 2, y = -6$$

From (ii), $x = 3 - 2 = 1$ or $x = 3 + 6 = 9$

$\therefore$ Required points of intersection are $(1, 2)$, $(9, -6)$

43. (b) :



44. (d) : $\int_{-6}^2 (3 - y) dy = \left[3y - \frac{y^2}{2} \right]_{-6}^2$

$$= \left[6 - \frac{4}{2} - \left[3(-6) - \frac{(-6)^2}{2} \right] \right] = 4 + 36 = 40$$

$$\begin{aligned}
 \text{45. (c): Required area} &= \int_{-6}^2 (3-y) dy - \int_{-6}^2 \frac{y^2}{4} dy \\
 &= 40 - \frac{1}{4} \left[\frac{y^3}{3} \right]_{-6}^2 = 40 - \frac{1}{4} \left[\frac{8}{3} - \frac{(-6)^3}{3} \right] \\
 &= 40 - \frac{2}{3} - \frac{216}{12} = \frac{480-8-216}{12} = \frac{256}{12} = \frac{64}{3} \text{ sq. units}
 \end{aligned}$$

$$\begin{aligned}
 \text{46. (b): Given: Let } I &= \int x e^{9x^2} dx \\
 \text{Put } 9x^2 = t &\Rightarrow 18x dx = dt \\
 \therefore I &= \int e^t \frac{dt}{18} = \frac{1}{18} e^t + C = \frac{e^{9x^2}}{18} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{47. (c): Given: } \int \frac{1}{x^2+64} dx &= a \tan^{-1} \left(\frac{x}{b} \right) + C \\
 &= \int \frac{1}{x^2+8^2} dx = \frac{1}{8} \tan^{-1} \left(\frac{x}{8} \right) + C \Rightarrow a = \frac{1}{8}, b = 8 \\
 \therefore a+b &= \frac{1}{8} + 8 \Rightarrow a+b = \frac{65}{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{48. (a): } \int \left(3\sqrt{x} + \frac{4}{x^2} + 6 \right) dx &= 64 \\
 \Rightarrow \int 3x^{1/2} dx + \int \frac{4}{x^2} dx + \int 6 dx &= 64
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \frac{3x^{1/2+1}}{\frac{1}{2}+1} + \frac{4x^{-2+1}}{-2+1} + 6x + C &= 64 \\
 \Rightarrow 2x^{\frac{3}{2}} - \frac{4}{x} + 6x + C &= 64
 \end{aligned}$$

At $x = 4$, we get

$$2(4)^{3/2} - \frac{4}{4} + 6(4) + C = 64 \Rightarrow C = 25$$

$$\begin{aligned}
 \text{49. (d): By the first fundamental theorem of Integral} \\
 \text{calculus } F'(t) &= \frac{d}{dt} \int_0^t (4x^6 + 1) dx = 4t^6 + 1
 \end{aligned}$$

$$\text{50. (a): } \frac{2x-3}{(x+2)(x-3)} = \frac{A}{(x+2)} + \frac{B}{(x-3)}$$

$$(2x-3) = A(x-3) + B(x+2) \quad \dots(i)$$

Putting $x = 3$ in (i), we get

$$2(3) - 3 = B(3+2) \Rightarrow 6 - 3 = 5B \Rightarrow B = \frac{3}{5}$$

Putting $x = -2$ in (i), we get

$$2(-2) - 3 = A(-2-3) = -5A \Rightarrow A = \frac{7}{5}$$

$$\therefore A + B = \frac{7}{5} + \frac{3}{5} = 2$$



In DU, switch from cut-off to CUET helps Bihar state board, hurts Kerala

A switch in the admission procedure in Delhi University (DU) this year from cut-offs to the Common University Entrance test (CUET) appears to have helped students from some state boards more than others. For instance, the percentage of students admitted to DU from the Bihar state board has shown an uptick, while for the Kerala state board, the figure shows a decline, data accessed by The Indian Express show.

With the adoption of CUET and a centralised allocation admission system, DU this year moved away from a decades-long system of admissions based on Class 12 marks. The push came after last year when a university panel, constituted to examine admission related issues, had flagged unequal admission opportunities to students of different boards.

Admission data from the first two rounds of seat allocations - in which 62,825 admissions

have taken place for roughly 70,000 seats in DU - shows that the bulk of admissions (85.67%) are still students under the two major national boards - CBSE (51,797 admissions) and CISCE (2,026 admissions). This is almost the same as last year's figure, in which these two boards constituted 85.95% of the final 71,748 admissions.

CHANGING STUDENT PATTERN IN D.U.

TOP 5 BOARDS (2022)	TOP 5 BOARDS (2021)
1. CBSE 51,797	1. CBSE 59,199
2. CISCE 2,026	2. Haryana 2,470
3. Bihar 1,450	3. CISCE 2,389
4. Uttar Pradesh 1,133	4. Kerala 1,672
5. Rajasthan 840	5. Rajasthan 1,511

The share in admissions of both these boards is also largely proportionate to the number of candidates who had registered - CBSE candidates constituted 81.34% of the applicants while CISCE candidates made up 3.87% of the applicants.

However, differences from last year begin to emerge when one looks at students from state boards. Last year, candidates from the Kerala board made up 2.33% of the total admissions that took place in DU. This year, their share in admissions has come down to 0.62%. Similarly, last year candidates from the Haryana board made up 3.44% of admissions. This figure has drastically come down to 0.77%.

It's a different story for candidates from the Bihar state board. Last year, they made up 0.77% of the admissions, while this year that figure is 2.3%.

Since the total number of applications this year (1.75 lakh) is considerably lower than last year (2.74 lakh), the acceptance rate - or the number of students who applied from a particular board and got admission - is generally higher, with one notable exception. Last year, the Kerala board had the highest acceptance rate among all major boards at 41.98%. This year, the figure has dropped to 21.17%.

Courtesy : The Indian Express

Are you ready for Olympiads?

Exam on

13th Dec 2022



CLASS XI

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Sets, Relations and Functions, Principle of Mathematical Induction, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequations, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Mathematical Reasoning, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Probability.

Practice Questions

- Which of the following is true?
A. $A \cup (B - C) = (A \cup B) \cap (A \cup C)$
B. $A \cup (B - C) = (A \cup B) - C$
C. $A \cap (B - C) = (A \cap B) - C$
D. $A \cap (B - C) = (A \cap B) - (A \cap C)$
- Find the domain and range respectively of the real function, $f(x) = \frac{1}{\sqrt{x-|x|}}$.
A. R^+ and $(0, \infty)$ B. ϕ, ϕ
C. R^- and $(0, \infty)$ D. None of these
- Solve $2\log_x a + \log_{ax} a + 3\log_b a = 0$, where $a > 0$, $b = a^2x$.
A. $a^{-1/2}$ B. $a^{-4/3}$
C. Both (A) and (B) D. None of these
- If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is
A. 10 B. 4 C. 2 D. 8
- If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to
A. 1 B. -1 C. 2 D. -2
- The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} \geq 0$ is
A. $(-\infty, -4) \cup [-3, 3] \cup (4, \infty)$
B. $(-\infty, -4) \cup (4, \infty)$
C. $(-\infty, -3) \cup (4, \infty)$
D. $(-\infty, -3) \cup (3, \infty)$
- If $2p + 3q + 4r = 15$, where $p, q, r > 0$, then the maximum value of $p^3q^5r^7$ is
A. 2180 B. $\frac{5^4 \cdot 3^5}{2^{15}}$ C. $\frac{5^5 \cdot 7^7}{2^{17} \cdot 9}$ D. 2285
- If x and y are acute angles such that $\sin x + \sin y = \frac{3}{4}$ and $\cos x + \cos y = \frac{3}{2}$, then the value of $\sin(x+y)$ is
A. 0.8 B. 1.2 C. 1.8 D. 0.5
- Find the equation of the straight line which passes through the origin and making angle 60° with the line $x + \sqrt{3}y + 3\sqrt{3} = 0$.
A. $x - \sqrt{3}y = 0$ B. $x = 0$
C. Both (A) and (B) D. None of these
- The centres of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$ are

- A. $(-10 \pm 3\sqrt{6}, 0)$ B. $(0, -10 \pm 3\sqrt{6})$
 C. $(0, 10 \pm 6\sqrt{2})$ D. $(10 \pm 6\sqrt{2}, 0)$

11. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

- A. 360 B. 390
 C. 780 D. None of these

12. Find the value of $\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \times 6 - {}^8C_3 \times 6^2 + \dots + {}^8C_8 \times 6^7$.

- A. 0 B. 2^8 C. 6^8 D. $5^8/6$

13. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5 were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is

- A. 8.00 B. 8.25 C. 9.00 D. 8.50

14. Evaluate: $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x - 2} - \sqrt{x + 2}}$

- A. 4 B. 8
 C. 16 D. None of these

15. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that all the three balls are white.

- A. $5/143$ B. $6/143$
 C. $5/13$ D. None of these

ACHIEVERS SECTION

16. Match column I with column II and select the correct option.

Column I		Column II	
(P)	The domain of the function $f(x) = \sqrt{4x - 3} + \sqrt{2x - 6}$ is	(1)	$\left[0, \frac{1}{2}\right]$
(Q)	The range of the function $y = \frac{x^2}{1 + x^4}$ is	(2)	$[3, \infty)$
(R)	The domain of $f(x) = \frac{\log_2(x + 3)}{x^2 + 3x + 2}$ is	(3)	$\left(-\infty, \frac{2}{3}\right] \cup (1, \infty)$

(S)	The range of $f(x) = \frac{x^2 - 2}{x^2 - 3}$ is	(4)	$(-3, \infty)$
-----	--	-----	----------------

- A. P-1, Q-2, R-4, S-3 B. P-3, Q-4, R-2, S-1
 C. P-4, Q-1, R-2, S-3 D. P-2, Q-1, R-4, S-3

17. Which of the following statements is incorrect.

- A. Number of rectangles on a chess board is ${}^4C_2 \times {}^4C_2$.

- B. If a, b, c, d, e are five positive numbers, then $\frac{b}{a} + \frac{c}{b} + \frac{d}{c} + \frac{e}{d} + \frac{a}{e} \geq 5$.

- C. The sum of an infinite G.P. is given by $\frac{a}{1 - r}$, where $|r| < 1$, 'a' is the first term and r is the common ratio.

- D. If $AB + BC = AC$, then ΔABC can not be possible.

18. **Statement-I**: The derivative of $h(x) = \frac{x + \cos x}{\tan x}$ is $\frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{(\tan x)^2}$.

Statement-II: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{(v)^2}$.

- A. Both Statement-I and Statement-II are true.
 B. Statement-I is true but Statement-II is false.
 C. Statement-I is false but Statement-II is true.
 D. Both Statement-I and Statement-II are false.

19. Which of the following is true?

- (i) The probability of drawing either an ace or a king from a pack of 52 cards in a single draw is $2/13$.

- (ii) If A and B are two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, then $\frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$.

- (iii) If A and B be mutually exclusive events in a sample space such that $P(A) = 0.3$ and $P(B) = 0.6$, then $P(\overline{A} \cap \overline{B}) = 0.28$.

- (iv) If A and B are mutually exclusive events, then $P(A \cap B) \neq 0$.

- A. (i), (ii) B. (i), (iii)
 C. (iii), (iv) D. All of these

20. If $f'(4) = 5$, $g'(4) = 12$, $f(4)g(4) = 2$ and $g(4) = 6$, then $\left(\frac{f}{g}\right)'(4) =$

- A. $5/36$ B. $11/18$ C. $23/36$ D. $13/18$

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (D) : We have, $A \cup (B - C) = (A \cup B) \cap (A \cup C')$
and $A \cap (B - C) = (A \cap B) - (A \cap C)$

2. (B) : Here, $f(x) = \frac{1}{\sqrt{x-|x|}}$

Since, we know that $|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$

$$\therefore x - |x| = \begin{cases} x - x, & \text{when } x \geq 0 \\ x + x, & \text{when } x < 0 \end{cases}$$

$$\Rightarrow x - |x| = \begin{cases} 0, & \text{when } x \geq 0 \\ 2x, & \text{when } x < 0 \end{cases}$$

$$\Rightarrow x - |x| \leq 0 \text{ for all } x \in \mathbb{R}$$

$$\Rightarrow \frac{1}{\sqrt{x-|x|}} \text{ is not defined for any } x \in \mathbb{R}$$

$\therefore$ Domain of $f(x) = \phi$ and Range of $f(x) = \phi$.

3. (C) : The given equation can be written as

$$\frac{2}{\log_a x} + \frac{1}{\log_a ax} + \frac{3}{\log_a a^2 x} = 0$$

$$\Rightarrow \frac{2}{\log_a x} + \frac{1}{\log_a a + \log_a x} + \frac{3}{2\log_a a + \log_a x} = 0$$

$$\Rightarrow \frac{2}{t} + \frac{1}{1+t} + \frac{3}{2+t} = 0, \text{ where } t = \log_a x$$

$$\Rightarrow 2(1+t)(2+t) + 3t(1+t) + t(2+t) = 0$$

$$\Rightarrow 6t^2 + 11t + 4 = 0$$

$$\Rightarrow (2t+1)(3t+4) = 0 \Rightarrow t = -\frac{1}{2} \text{ or } -\frac{4}{3}$$

$$\Rightarrow \log_a x = -\frac{1}{2} \text{ or } \log_a x = -\frac{4}{3}$$

$$\therefore x = a^{-1/2} \text{ or } x = a^{-4/3}.$$

4. (C) : $|z_1| = 1$

$$\Rightarrow z_1 \bar{z}_1 = 1, |z_2| = 2 \Rightarrow z_2 \bar{z}_2 = 4, |z_3| = 3 \Rightarrow z_3 \bar{z}_3 = 9.$$

$$\text{Also, } |9z_1 z_2 + 4z_1 z_3 + z_2 z_3| = 12$$

$$\Rightarrow |z_1 z_2 z_3 \bar{z}_3 + z_1 z_2 z_3 \bar{z}_2 + z_1 \bar{z}_1 z_2 z_3| = 12$$

$$\Rightarrow |z_1 z_2 z_3| |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$$

$$\Rightarrow |z_1| |z_2| |z_3| |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12 \Rightarrow 6 |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$$

$$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 2 \Rightarrow |z_1 + z_2 + z_3| = 2$$

5. (C) : $x_1 + x_2 = -\frac{b}{a}, x_1 x_2 = \frac{c}{a}$

$$\text{Now } \frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{x_1^2 + x_2^2}{x_1^2 x_2^2}$$

$$= \frac{(x_1 + x_2)^2 - 2x_1 x_2}{(x_1 x_2)^2} = \left(\frac{x_1 + x_2}{x_1 x_2} \right)^2 - \frac{2}{x_1 x_2} = \frac{b^2}{c^2} - \frac{2a}{c}$$

$$\text{We have, } \frac{b^2}{c^2} - \frac{2a}{c} = -\frac{b}{a} \Rightarrow \frac{b^2}{c} - \frac{2ac}{c} = -\frac{bc}{a}$$

$$\Rightarrow \frac{b^2}{ac} - 2 = -\frac{bc}{a^2} \Rightarrow \frac{b^2}{ac} + \frac{bc}{a^2} = 2$$

6. (A) : Case I : When $x \geq 0$

$$\therefore |x| = x$$

$$\Rightarrow \frac{3-|x|}{4-|x|} = \frac{3-x}{4-x} \geq 0 \Rightarrow \frac{(3-x)(4-x)}{(4-x)^2} \geq 0$$

$$\Rightarrow (x-3)(x-4) \geq 0 \text{ and } x \neq 4 \Rightarrow x \leq 3 \text{ or } x > 4$$

$$\text{But } x \geq 0, \text{ so } x \in [0, 3] \cup (4, \infty) \quad \dots(i)$$

Case II : When $x < 0$

$$\therefore |x| = -x$$

$$\Rightarrow \frac{3-|x|}{4-|x|} = \frac{3+x}{4+x} \geq 0 \Rightarrow \frac{(x+3)(x+4)}{(x+4)^2} \geq 0$$

$$\Rightarrow (x+3)(x+4) \geq 0 \text{ and } x \neq -4 \Rightarrow x < -4 \text{ or } x \geq -3$$

$$\text{But } x < 0 \text{ so, } x \in (-\infty, -4) \cup [-3, 0) \quad \dots(ii)$$

From (i) and (ii), x lies in the interval

$$(-\infty, -4) \cup [-3, 3] \cup (4, \infty).$$

7. (C) : The given sum can be written as

$$\frac{2p}{3} + \frac{2p}{3} + \frac{2p}{3} + \underbrace{\frac{3q}{5} + \dots + \frac{3q}{5}}_{5 \text{ times}} + \underbrace{\frac{4r}{7} + \dots + \frac{4r}{7}}_{7 \text{ times}}$$

$$\text{Now, } \frac{15}{15} \geq \sqrt[15]{\left(\frac{2p}{3}\right)^3 \left(\frac{3q}{5}\right)^5 \left(\frac{4r}{7}\right)^7} \quad (\because \text{A.M.} \geq \text{G.M.})$$

$$\Rightarrow p^3 q^5 r^7 \frac{2^3 3^5 4^7}{3^3 5^5 7^7} \leq 1 \Rightarrow p^3 q^5 r^7 \leq \frac{5^5 7^7}{2^3 3^2 4^7}$$

$$8. \text{ (A) : We have, } \sin x + \sin y = \frac{3}{4} \quad \dots(i)$$

$$\text{and } \cos x + \cos y = \frac{3}{2} \quad \dots(ii)$$

Dividing (i) by (ii), we get

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)}{2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)} = \frac{1}{2} \Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\text{Now, } \sin(x+y) = \frac{2 \tan\left(\frac{x+y}{2}\right)}{1 + \tan^2\left(\frac{x+y}{2}\right)} = \frac{2 \times \frac{1}{2}}{1 + \frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = 0.8.$$

$$9. \text{ (C) : Given line is } x + \sqrt{3}y + 3\sqrt{3} = 0 \quad \dots(i)$$

$$\Rightarrow y = \left(-\frac{1}{\sqrt{3}}\right)x - 3 \quad \therefore \text{Slope of (i)} = -\frac{1}{\sqrt{3}}.$$

Let slope of the required line be m .

$$\therefore \tan 60^\circ = \left| \frac{m - (-1/\sqrt{3})}{1 + m(-1/\sqrt{3})} \right| \Rightarrow \sqrt{3} = \left| \frac{\sqrt{3}m + 1}{\sqrt{3} - m} \right|$$

$$\Rightarrow \frac{\sqrt{3}m + 1}{\sqrt{3} - m} = \pm \sqrt{3}$$

$$\text{Now, if } \frac{\sqrt{3}m + 1}{\sqrt{3} - m} = \sqrt{3} \Rightarrow \sqrt{3}m + 1 = 3 - \sqrt{3}m \Rightarrow m = \frac{1}{\sqrt{3}}$$

Using $y = mx + c$, the equation of the required line is

$$y = \frac{1}{\sqrt{3}}x \Rightarrow x - \sqrt{3}y = 0$$

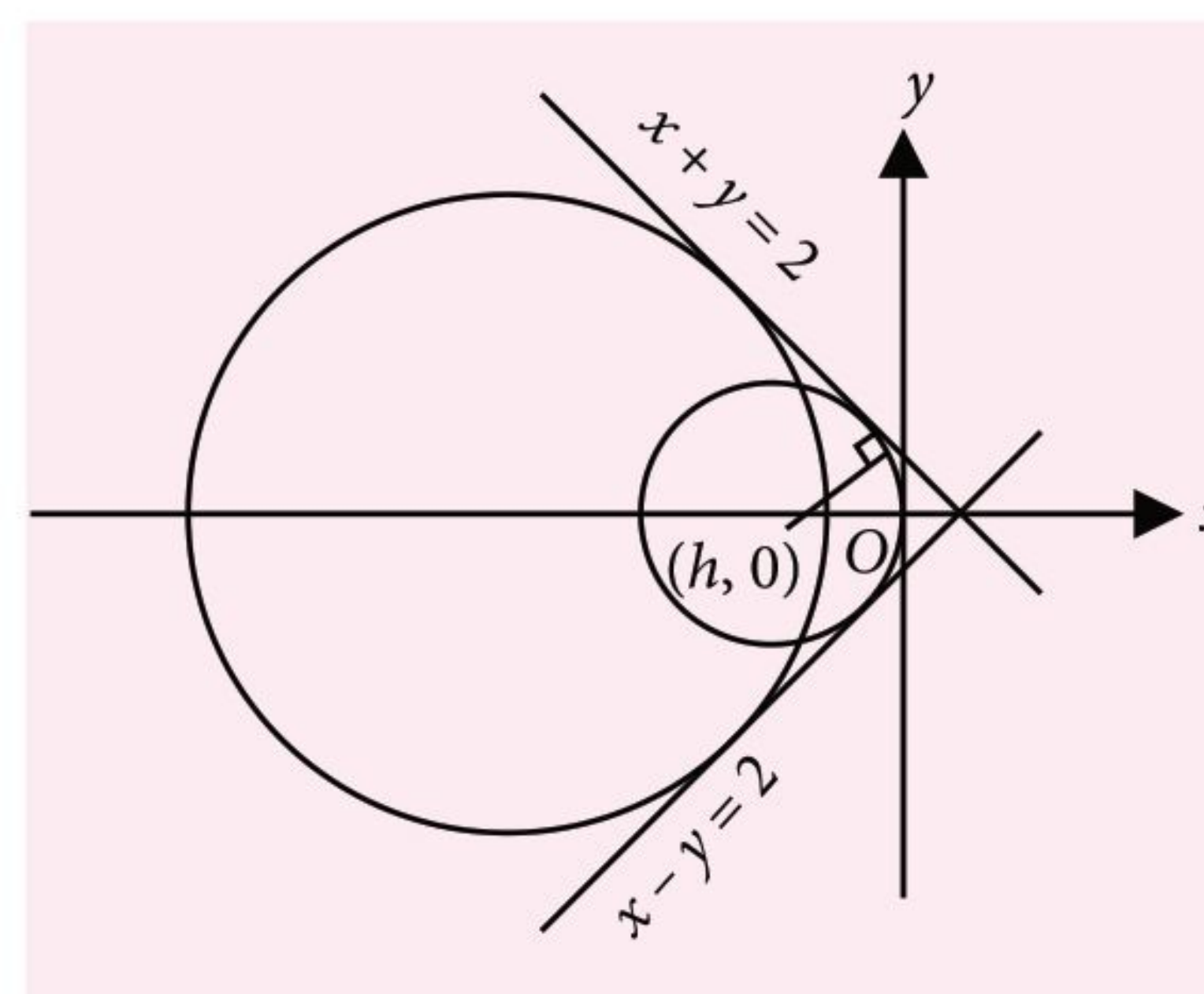
$$\text{If } \frac{\sqrt{3}m + 1}{\sqrt{3} - m} = -\sqrt{3} \Rightarrow \sqrt{3}m + 1 = -3 + \sqrt{3}m,$$

where m is not defined.

$\therefore$ The slope of the required line is not defined. Thus, the required line is a vertical line. This line is to pass through the origin.

$\therefore$ The equation of the required line is $x = 0$.

10. (A) : Since the circle touches both the lines $x + y = 2$ and $x - y = 2$, its centre must lie on the x -axis. Let the centre of the circle be $(h, 0)$.



Now, the radius of the circle is equal to the perpendicular distance of the point $(h, 0)$ from the line $x + y - 2 = 0$, i.e.,

$$\text{Radius} = \frac{|h + 0 - 2|}{\sqrt{2}} = \frac{|h - 2|}{\sqrt{2}}$$

Then, the equation of the circle is

$$(x - h)^2 + (y - 0)^2 = \frac{(h - 2)^2}{2}$$

Since the circle passes through the point $(-4, 3)$, we have

$$(-4 - h)^2 + (3 - 0)^2 = \frac{(h - 2)^2}{2} \Rightarrow h^2 + 20h + 46 = 0$$

$$\Rightarrow h = \frac{-20 \pm \sqrt{400 - 184}}{2} = -10 \pm 3\sqrt{6}$$

11. (C) : Since, candidate cannot attempt more than 5 questions from either group.

Thus, he has to attempt minimum two questions from other group.

The number of questions that can be attempted from each group is given in following table

Group I	5	4	3	2
Group II	2	3	4	5

Hence, total number of possible ways

$$= {}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_2 \times {}^6C_5$$

$$= 2[{}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3] = 2[90 + 300] = 2 \times 390 = 780$$

$$12. \text{ (D) : Clearly, } \frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \times 6 - {}^8C_3 \times 6^2 + \dots + {}^8C_8 \times 6^7$$

$$= \frac{1}{6}[{}^8C_0 - {}^8C_1(6) + {}^8C_2(6)^2 - {}^8C_3(6)^3 + \dots + {}^8C_8 6^8]$$

$$= \frac{1}{6}(1 - 6)^8$$

$$[\because (1 - x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \dots + (-1)^n {}^nC_nx^n]$$

$$= \frac{1}{6} \times 5^8 = \frac{5^8}{6}.$$

13. (C) : Incorrect $\sum_{i=1}^{100} x_i = 400$

and incorrect $\sum_{i=1}^{100} x_i^2 = 2475$

Now incorrect observations 3, 4 and 5 are omitted.

$\therefore$ Correct $\sum_{i=1}^{97} x_i = 400 - 3 - 4 - 5 = 388$

and correct $\sum_{i=1}^{97} x_i^2 = 2475 - 3^2 - 4^2 - 5^2$
 $= 2475 - 50 = 2425$

Now, Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N} \right)^2$

$$= \frac{2425}{97} - \left(\frac{388}{97} \right)^2 = 25 - 16 = 9$$

14. (B) : We have, $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{3x-2} - \sqrt{x+2}}$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{3x-2} + \sqrt{x+2})}{(\sqrt{3x-2} - \sqrt{x+2})(\sqrt{3x-2} + \sqrt{x+2})}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{3x-2} + \sqrt{x+2})}{(\sqrt{3x-2})^2 - (\sqrt{x+2})^2}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{3x-2} + \sqrt{x+2})}{(3x-2) - (x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{3x-2} + \sqrt{x+2})}{2x - 4}$$

$$= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)(\sqrt{3x-2} + \sqrt{x+2})}{2(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{3x-2} + \sqrt{x+2})}{2}$$

$$= \frac{(2+2)(\sqrt{6-2} + \sqrt{2+2})}{2} = \frac{4(2+2)}{2} = 8$$

15. (A) : Number of red balls = 8

and number of white balls = 5

Total number of balls = 13

$$P(\text{all the three balls are white}) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{5}{143}$$

16. (D) : (P) Here, $f(x) = \sqrt{4x-3} + \sqrt{2x-6}$

Clearly, $f(x)$ is defined when $4x-3 \geq 0$ and $2x-6 \geq 0$

$$\Rightarrow 4x \geq 3 \text{ and } 2x \geq 6$$

$$\Rightarrow x \geq 3/4 \text{ and } x \geq 3 \Rightarrow x \in [3, \infty)$$

(Q) Here, $y = f(x) = \frac{x^2}{1+x^4}$

Clearly, $\frac{x^2}{1+x^4} \geq 0$ for any $x \in R$.

$$\text{Now, } (x^2 - 1)^2 \geq 0 \Rightarrow x^4 + 1 - 2x^2 \geq 0 \Rightarrow x^4 + 1 \geq 2x^2$$

$$\Rightarrow \frac{x^2}{1+x^4} \leq \frac{x^2}{2x^2} = \frac{1}{2} \quad \therefore \text{Range of } f(x) = \left[0, \frac{1}{2} \right]$$

(R) Here $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$

Clearly, $f(x)$ is defined if $(x+3) > 0$ i.e., $x > -3$

$$\text{and } x^2 + 3x + 2 \neq 0 \Rightarrow (x+1)(x+2) \neq 0$$

$$\Rightarrow x \neq -1 \text{ or } x \neq -2$$

$$\therefore \text{Domain of } f(x) = (-3, \infty) - \{-1, -2\}.$$

(S) Let $y = f(x) = \frac{x^2-2}{x^2-3} \Rightarrow y(x^2-3) = x^2-2$

$$\Rightarrow yx^2 - 3y = x^2 - 2 \Rightarrow x^2 = \frac{3y-2}{y-1} \geq 0$$

$$\therefore y \in \left(-\infty, \frac{2}{3} \right] \cup (1, \infty)$$

$$(P) \rightarrow 2, (Q) \rightarrow 1, (R) \rightarrow 4, (S) \rightarrow 3$$

17. (A)

A. In a chess board, there are 9 horizontal lines & 9 vertical lines.

$\therefore$ Total number of rectangles of any size are ${}^9C_2 \times {}^9C_2$.

B. Given, a, b, c, d, e are five positive numbers.
 Using relationship between A.M and G.M, on



**START THE JOURNEY TOWARDS
IMO OLYMPIAD SUCCESS NOW!**

Download Previous 5 Years' Papers!

CLASS

11



Scan & Download



**DOWNLOAD
NOW**

$\frac{b}{a}, \frac{c}{b}, \frac{d}{c}, \frac{e}{d}, \frac{a}{e}$, we have

$$\frac{\frac{b}{a} + \frac{c}{b} + \frac{d}{c} + \frac{e}{d} + \frac{a}{e}}{5} \geq \left(\frac{b}{a} \times \frac{c}{b} \times \frac{d}{c} \times \frac{e}{d} \times \frac{a}{e} \right)^{1/5}$$

$$\Rightarrow \frac{b}{a} + \frac{c}{b} + \frac{d}{c} + \frac{e}{d} + \frac{a}{e} \geq 5$$

(C) Sum of infinite G.P. = $\frac{a}{1-r}$, $|r| < 1$.

(D) As $AB + BC = AC$

$\Rightarrow A, B, C$ are collinear $\Rightarrow \triangle ABC$ cannot be possible.

18. (A) : Statement-I : We have, $h(x) = \frac{x + \cos x}{\tan x}$... (i)

Differentiating (i) both sides w.r.t. 'x', we get

$$h'(x) = \frac{(x + \cos x)' \tan x - (x + \cos x)(\tan x)'}{(\tan x)^2}$$

$$= \frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{(\tan x)^2}, \text{ which is true.}$$

Also, statement-II is true.

19. (A) : (i) Required probability = $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$.

(ii) $P(A \cup B) \geq \max \{P(A), P(B)\} = \frac{2}{3}$ or $P(A \cup B) \geq \frac{2}{3}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$

$$= \frac{1}{2} + \frac{2}{3} - 1 = \frac{1}{6}$$

$$\text{or } P(A \cap B) \geq \frac{1}{6} \quad \dots (i)$$

$$\text{and } P(A \cap B) \leq \min \{P(A), P(B)\} = \frac{1}{2}$$

$$\Rightarrow P(A \cap B) \leq \frac{1}{2} \quad \dots (ii)$$

From (i) and (ii), we get $\frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$

(iii) $\because A$ and B are mutually exclusive events

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 0 = 0.3 + 0.6 = 0.9$$

$$\therefore P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - 0.9 = 0.1$$

(iv) For mutually exclusive, $A \cap B = \phi \therefore P(A \cap B) = 0$

20. (D) : We know that

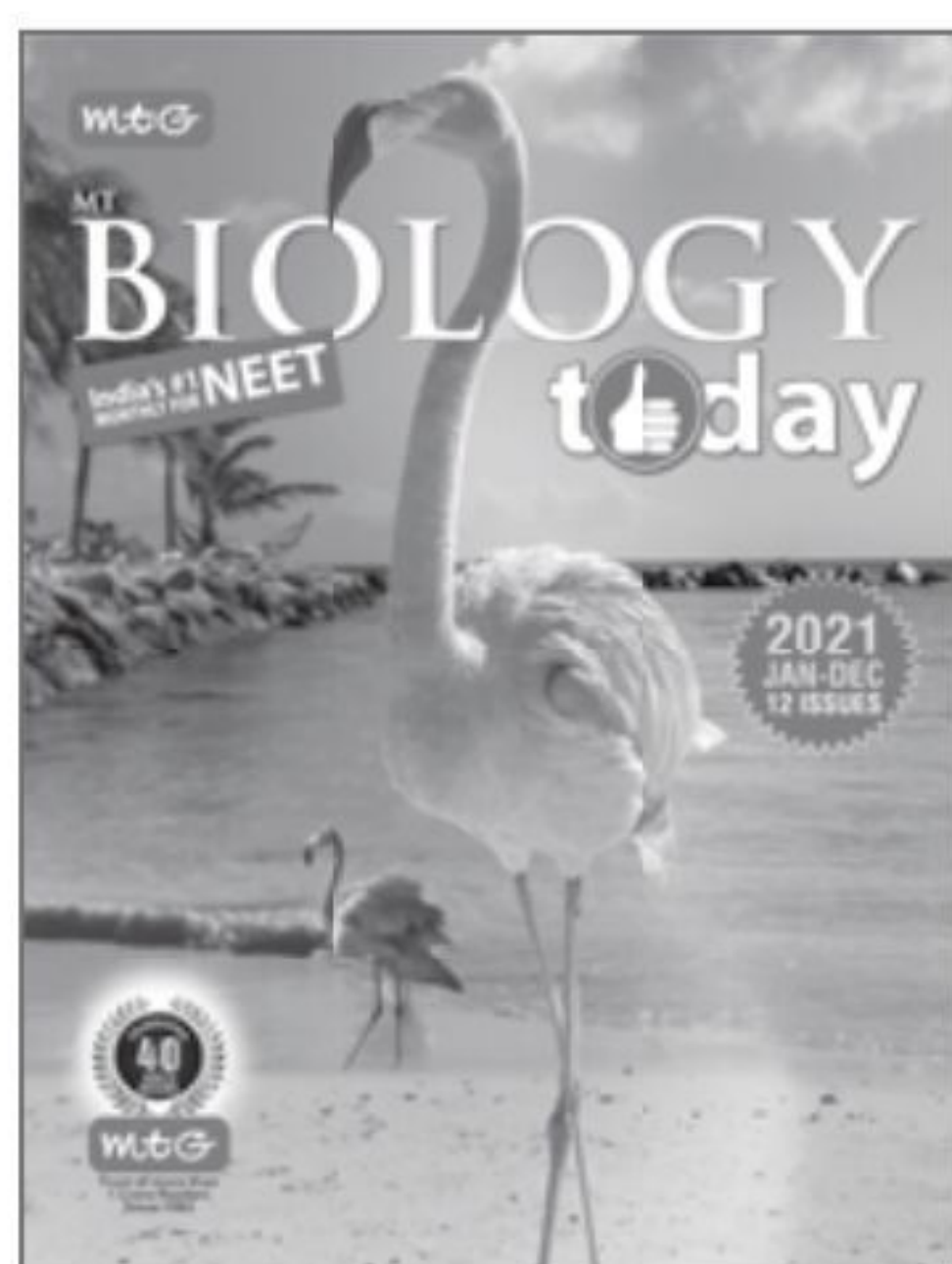
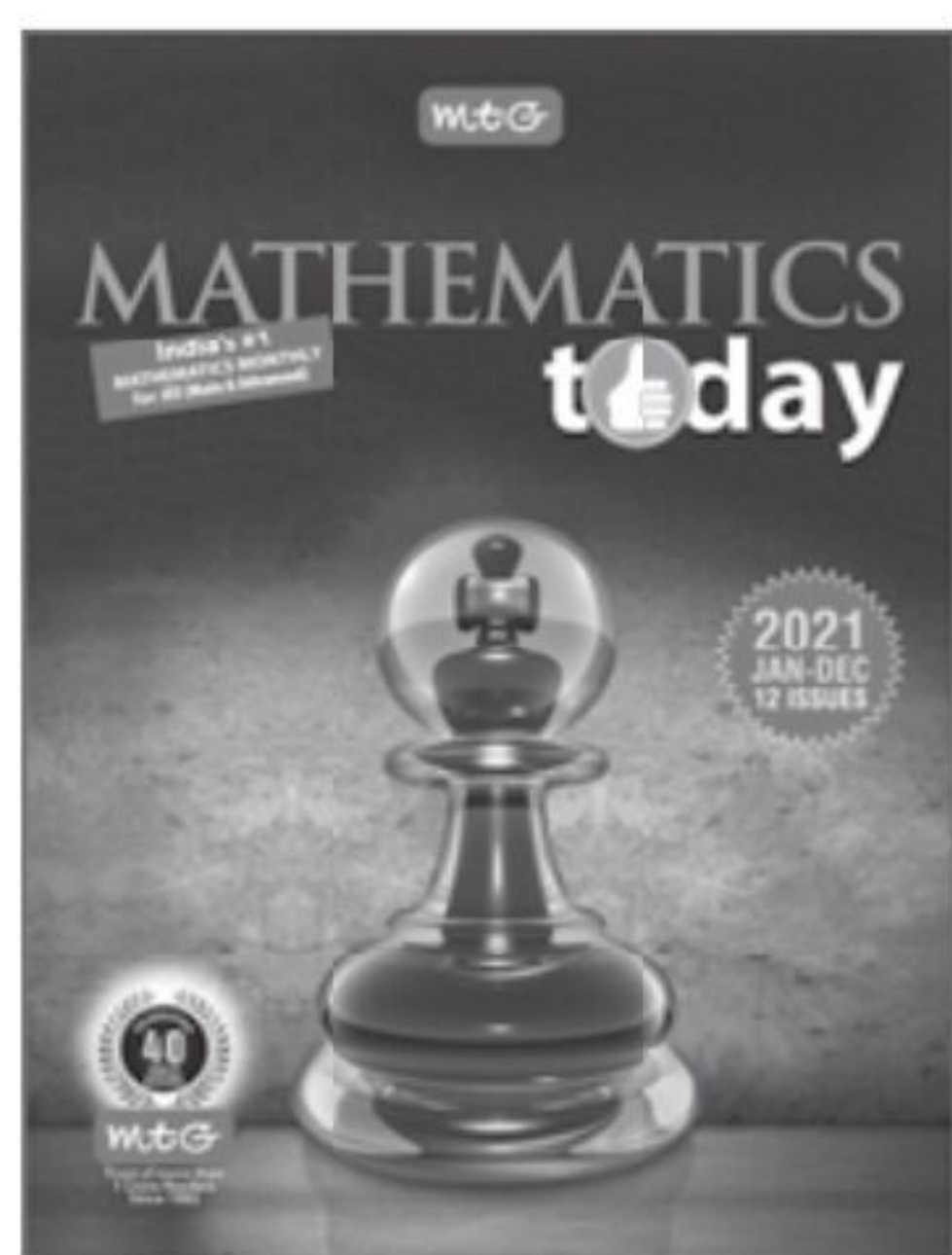
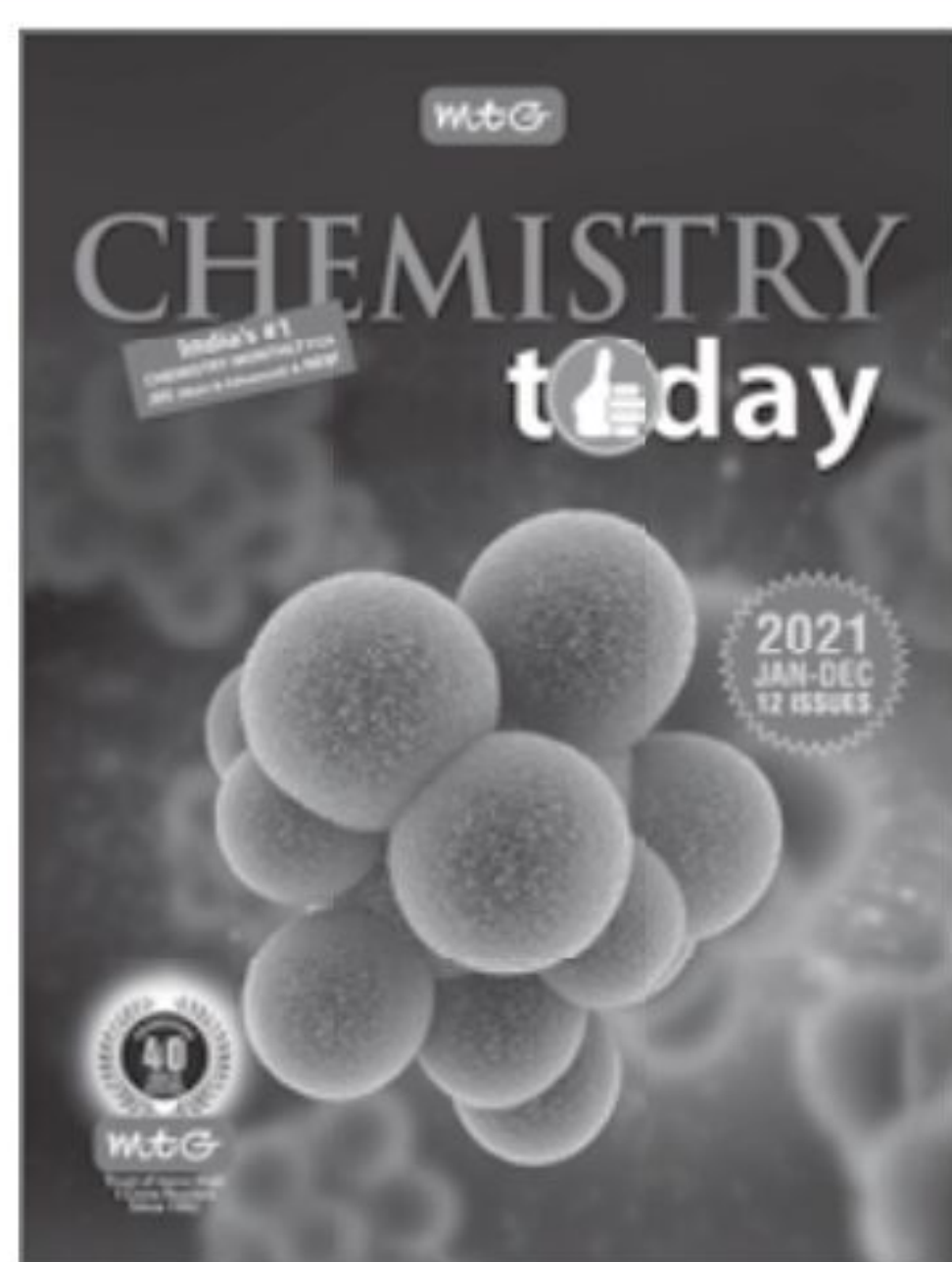
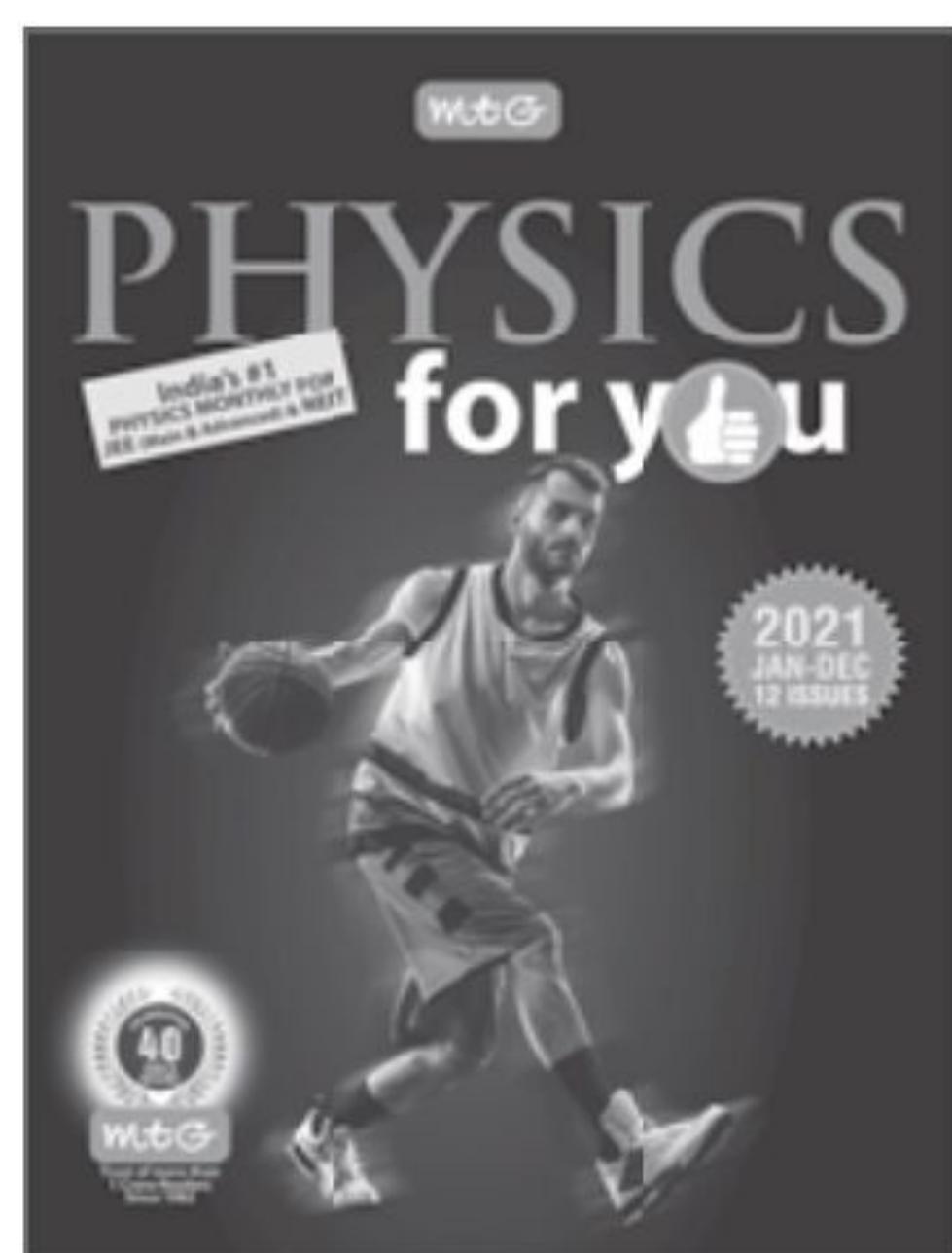
$$\left(\frac{f}{g} \right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\therefore \left(\frac{f}{g} \right)'(4) = \frac{g(4) \cdot f'(4) - f(4) \cdot g'(4)}{[g(4)]^2} = \frac{6 \times 5 - \frac{1}{3} \times 12}{36} = \frac{13}{18}$$

For other sections/subjects please refer to
Physics For You, Chemistry Today and Biology Today



AVAILABLE BOUND VOLUMES



buy online at www.mtg.in

2021

Physics For You (January - December)	₹ 380 12 issues
Mathematics Today (January - December)	₹ 380 12 issues
Biology Today (January - December)	₹ 380 12 issues

2019

Mathematics Today (January - December)	₹ 380 12 issues
---	--------------------

2018

Physics For You (January - December)	₹ 380 12 issues
Chemistry Today (January - December)	₹ 380 12 issues
Mathematics Today (January - December)	₹ 380 12 issues
Biology Today (January - December)	₹ 380 12 issues

of your favourite magazines

How to order : Send money by demand draft/money order. Demand Draft should be drawn in favour of **MTG Learning Media (P) Ltd.** Mention the volume you require along with your name and address. OR buy online from www.mtg.in

Add ₹ 90 as postal charges

Older issues can be accessed on **digital.mtg.in** in digital form.

Mail your order to :

Circulation Manager,
MTG Learning Media (P) Ltd.
Plot No. 99, Sector 44
Institutional Area, Gurugram, (HR)
Tel.: (0124) 6601200
E-mail : info@mtg.in
Web : www.mtg.in

Are you ready for Olympiads?

Exam on

13th Dec 2022



CLASS XII

SYLLABUS*

Section – 1 : Verbal and Non-Verbal Reasoning.

Section – 2 : Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

OR

Section – 2 : Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

Section – 3 : The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

Section – 4 : Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50

Time : 1 hr.

PATTERN & MARKING SCHEME

Section	(1) Logical Reasoning	(2) Mathematical Reasoning or Applied Mathematics	(3) Everyday Mathematics	(4) Achievers Section
No. of Questions	15	20	10	5
Marks per Ques.	1	1	1	3

Practice Questions

1. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by

$R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by

- A. $(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)$
 B. $(2, 2), (3, 2), (4, 2), (2, 4)$
 C. $(3, 3), (4, 3), (5, 4), (3, 4)$
 D. None of these

2. Find the domain of $\sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$.

- A. $\left[-\frac{1}{2}, 1\right]$ B. $[-1, 1]$
 C. $\left[-\frac{1}{2}, \frac{1}{4}\right]$ D. $\left[-\frac{1}{4}, \frac{1}{2}\right]$

3. If matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for $n \geq 2$, A^n is equal to

- A. $2^{n-1}A - (n-1)I$ B. $2^{n-1}A - I$
 C. $nA - (n-1)I$ D. $nA - I$

4. Let k be a positive real number and

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $[k]$, where $[\cdot]$ is G.I.F, is

- A. 1 B. 4 C. 2 D. 3

5. If the function $f(x) = \begin{cases} \frac{e^{px} + \log_e(1+4x) + q}{x^3}, & x \neq 0 \\ r, & x = 0 \end{cases}$

is continuous at $x = 0$, then the value of $6pqr$ is

- A. 512 B. -512 C. -256 D. 256

6. If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, then $f'(0)$ is equal to

- A. $\left(2\log\frac{a}{b} + \frac{a^2-b^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$
 B. $\left(2\log\frac{a}{b} + \frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$
 C. $\left(2\log\frac{a}{b} + \frac{a^2-b^2}{ab}\right)\left(\frac{a}{b}\right)^{a-b}$
 D. None of these

7. Function $f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$ is monotonically increasing, if

- A. $\lambda > 1$ B. $\lambda < 1$ C. $\lambda < 4$ D. $\lambda > 4$

8. Evaluate : $\int \frac{x^x(x^{2x}+1)(1+\log x)}{x^{4x}+1} dx$

A. $\frac{1}{2} \tan^{-1} \left(\frac{x^{2x} + 1}{x^x \sqrt{2}} \right) + C$

B. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^{2x} - 1}{x^x \sqrt{2}} \right) + C$

C. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^{2x} + 1}{x^x \sqrt{2}} \right) + C$

D. $\frac{1}{2} \tan^{-1} \left(\frac{x^{2x} - 1}{x^x \sqrt{2}} \right) + C$

9. The area between two arms of the curve $|y| = x^3$ from $x = 0$ to $x = 2$ is

- A. 2 sq. units B. 4 sq. units
C. 8 sq. units D. 16 sq. units

10. Solve the initial value problem

$$xe^{y/x} - y + x \frac{dy}{dx} = 0, y(e) = 0.$$

- A. $y = -x \log(\log|x|); x \in (-\infty, -1) \cup (1, \infty)$
B. $y = \log(x), x \in (-\infty, -1) \cup (1, \infty)$
C. $y = x, x \in (-\infty, -1) \cup (1, \infty)$
D. $y = x + \log x, x \in (-\infty, -1) \cup (1, \infty)$

11. $\vec{a}, \vec{b}$ and $\vec{c}$ are perpendicular to $\vec{b} + \vec{c}, \vec{c} + \vec{a}$ and $\vec{a} + \vec{b}$ respectively and if $|\vec{a} + \vec{b}| = 6, |\vec{b} + \vec{c}| = 8$ and $|\vec{c} + \vec{a}| = 10$, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

- A. $5\sqrt{2}$ B. 50 C. $10\sqrt{2}$ D. 10

12. The shortest distance between the lines

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{-3} \text{ is}$$

- A. $\frac{372\sqrt{2}}{35}$ B. $\frac{186}{35}$ C. $\frac{372}{35\sqrt{2}}$ D. $\frac{186}{35\sqrt{2}}$

13. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATA NAGAR?

- A. 7/11 B. 6/11
C. 5/11 D. None of these

14. The maximum value of $Z = 4x + y$ subject to the constraints, $x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$ is

- A. 110 B. 50 C. 120 D. 130

15. $\int_{1/3}^3 \frac{1}{x} \log_e \left(\left| \frac{x+x^2-1}{x-x^2+1} \right| \right) dx$ is equal to

- A. 8/3 B. -8/3 C. 0 D. 3

ACHIEVERS SECTION

16. Read the given statement carefully and state 'T' for true and 'F' for false.

- (i) The value of the integral

$$\int_0^{2a} \left[\frac{f(x)}{\{f(x) + f(2a-x)\}} \right] dx \text{ is equal to } a.$$

- (ii) If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$,

then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ must be congruent.

- (iii) If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is 0.5.

- | | (i) | (ii) | (iii) |
|----|-----|------|-------|
| A. | T | T | T |
| B. | F | F | F |
| C. | T | F | F |
| D. | T | T | F |

17. Fill in the blanks and select the correct option.

- (i) A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of determinant chosen is positive is P.

- (ii) The integral $\int_0^{1.5} [x^2] dx$ where $[.]$ denotes the greatest integer function, equals Q.

- (iii) If two events A and B are such that $P(A^C) = 0.3, P(B) = 0.4$ and $P(A \cap B^C) = 0.5$, then $P(B/(A \cup B^C)) = \underline{R}$.

- | | P | Q | R |
|----|------|----------------|------|
| A. | 1/8 | 2 | 0.25 |
| B. | 3/16 | $2 - \sqrt{2}$ | 0.25 |
| C. | 1/4 | $\sqrt{2}$ | 0.3 |
| D. | 1/16 | 2 | 0.25 |

18. **Statement-1** : Area enclosed by the curve

$|x| + |y| = 2$ is 8 units.

Statement-2 : $|x| + |y| = 2$ represents a square of side of length $\sqrt{8}$ units.

- A. Statement-1 is true, Statement-2 is true.
B. Statement-1 is false, Statement-2 is false.
C. Statement-1 is true, Statement-2 is false.
D. Statement-1 is false, Statement-2 is true.



QR Code reader
app needed

Get a taste of competition early on

Inviting JEE aspirants to participate in
SOF International Mathematics Olympiad



SOF INTERNATIONAL MATHEMATICS OLYMPIAD

13 DEC. 2022

Millions of students compete in the SOF Olympiads.
Can there be a better, global, competitive platform
for you to assess your readiness for success in JEE?
To participate, all you need to do is contact your

School Principal or your school's
SOF Olympiads Coordinator immediately.
For more info, log on to sofworld.org.

*Top rank at the International level gets you a Gold medal and prize money of ₹ 50,000.
Top rank at the zonal level gets you a Gold medal and prize money of ₹ 5,000. Many more
prizes too, visit sofworld.org for details. Or just scan the QR code with your smartphone or tablet.*



Registrations closing soon. Hurry! Contact your Principal / School SOF Olympiads Coordinator today.

19. Match Column I with Column II and select the correct option.

Column I		Column II	
(P)	The maximum value of $f(x) = 9x(x-1)^2$, $0 \leq x \leq 2$ is	(1)	-1
(Q)	The minimum value of $f(x) = \frac{x^2-1}{x^2+1}$ is	(2)	18
(R)	The local maximum value of $f(x) = \frac{x^2+1}{x^2-1}$, $x \in R - \{-1, 1\}$ is	(3)	-2
(S)	The local maximum value of $f(x) = \frac{x^2+1}{x}$, $x \in R - \{0\}$ is	(4)	2

- A. P-3, Q-2, R-1, S-4 B. P-4, Q-3, R-2, S-1
C. P-2, Q-1, R-1, S-3 D. P-1, Q-4, R-3, S-2

20. The solution of the differential equation

$$x \frac{dy}{dx} - y = 10x^2 \text{ is}$$

- A. $\sin^{-1}\left(\frac{y}{x}\right) - 5x^2 = C$
B. $\sin^{-1}\left(\frac{y}{x}\right) = 10x^2 + C$
C. $\frac{y}{x} = 5x^2 + C$
D. $\sin^{-1}\left(\frac{y}{x}\right) = 10x^2 + Cx$

Darken your choice with HB Pencil

1. (A) (B) (C) (D)	5. (A) (B) (C) (D)	9. (A) (B) (C) (D)	13. (A) (B) (C) (D)	17. (A) (B) (C) (D)
2. (A) (B) (C) (D)	6. (A) (B) (C) (D)	10. (A) (B) (C) (D)	14. (A) (B) (C) (D)	18. (A) (B) (C) (D)
3. (A) (B) (C) (D)	7. (A) (B) (C) (D)	11. (A) (B) (C) (D)	15. (A) (B) (C) (D)	19. (A) (B) (C) (D)
4. (A) (B) (C) (D)	8. (A) (B) (C) (D)	12. (A) (B) (C) (D)	16. (A) (B) (C) (D)	20. (A) (B) (C) (D)

SOLUTIONS

1. (D) : We have, $(a, b) \in R \Leftrightarrow |a^2 - b^2| < 16$
 $\therefore a = 1 \Rightarrow |1 - b^2| < 16 \Rightarrow |b^2 - 1| < 16$
 $\Rightarrow -15 < b^2 < 17 \Rightarrow 0 < b^2 < 17 \Rightarrow b = 1, 2, 3, 4$
 $a = 2 \Rightarrow |4 - b^2| < 16 \Rightarrow |b^2 - 4| < 16$
 $\Rightarrow -12 < b^2 < 20$
 $\Rightarrow 0 < b^2 < 20 \Rightarrow b = 1, 2, 3, 4$
 $a = 3 \Rightarrow |9 - b^2| < 16 \Rightarrow |b^2 - 9| < 16$
 $\Rightarrow -7 < b^2 < 25$
 $\Rightarrow 0 < b^2 < 25 \Rightarrow b = 1, 2, 3, 4$
 $a = 4 \Rightarrow |16 - b^2| < 16 \Rightarrow |b^2 - 16| < 16$
 $\Rightarrow 0 < b^2 < 32 \Rightarrow b = 1, 2, 3, 4, 5$
 $a = 5 \Rightarrow |25 - b^2| < 16 \Rightarrow |b^2 - 25| < 16$
 $\Rightarrow 9 < b^2 < 41 \Rightarrow b = 4, 5$
 $\therefore R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 4), (5, 5)\}.$

2. (D) : We have, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$
For $f(x)$ to be defined, $\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$
 $\Rightarrow \sin^{-1}(2x) \geq -\pi/6$
But $-\frac{\pi}{2} \leq \sin^{-1}(\theta) \leq \frac{\pi}{2} \therefore -\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \frac{\pi}{2}$
 $\Rightarrow -\frac{1}{2} \leq 2x \leq 1 \Rightarrow -\frac{1}{4} \leq x \leq \frac{1}{2} \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

3. (C) : Given, $A^2 = 2A - I$
Now, $A^3 = A(A^2) = A(2A - I) = 2A^2 - A$
 $= 2(2A - I) - A = 3A - 2I$
Also, $A^4 = A(A^3) = A(3A - 2I) = 3A^2 - 2A$
 $= 3(2A - I) - 2A = 4A - 3I$
Continuing in this way, we get $A^n = nA - (n-1)I$
4. (B) : $\det A = (2k-1)(4k^2-1)$
 $+ 2\sqrt{k}(4k\sqrt{k} + 2\sqrt{k}) + 2\sqrt{k}(4k\sqrt{k} + 2\sqrt{k})$
 $= (2k-1)(4k^2-1) + 8(2k+1)k$
 $= (2k+1)((2k-1)^2 + 8k) = (2k+1)^3$
 $\det(\text{adj } A) = (\det A)^2 = (2k+1)^6$
 $\det B = 0$ [since B is skew symmetric matrix of order 3]
 $\det(\text{adj } B) = (\det B)^2 = 0$
 $\therefore (2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10 \Rightarrow k = 4.5$
So, $[k] = [4.5] = 4$

5. (D) : It is given that $f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0} \frac{e^{px} + \log_e(1+4x) + q}{x^3} = r$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\left\{ 1 + (px) + \frac{p^2 x^2}{2!} + \frac{p^3 x^3}{3!} + \dots + \left(4x - \frac{16}{2} x^2 + \frac{64}{3} x^3 + \dots \right) + q \right\}}{x^3} = r$$

$$\Rightarrow \lim_{x \rightarrow 0} \left\{ \frac{(1+q) + (p+4)x + \left(\frac{p^2}{2} - 8\right)x^2 + \left(\frac{p^3}{6} + \frac{64}{3}\right)x^3 + \dots}{x^3} \right\} = r$$

$$\therefore 1+q=0, p+4=0 \text{ and } \frac{p^2}{2} - 8 = 0 \text{ and } \frac{p^3}{6} + \frac{64}{3} = r$$

$$\Rightarrow q = -1, p = -4, r = \frac{32}{3}$$

$$\therefore 6pqr = 6 \times (-4) \times (-1) \times \frac{32}{3} = 256$$

6. (B)

7. (D) : We have, $f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$

On differentiating w.r.t. x , we get

$$f'(x) = \frac{[(2 \sin x + 3 \cos x)(\lambda \cos x - 6 \sin x) - (\lambda \sin x + 6 \cos x)(2 \cos x - 3 \sin x)]}{(2 \sin x + 3 \cos x)^2}$$

The given function is monotonically increasing, if $f'(x) > 0$

$$\Rightarrow 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) > 0$$

$$\Rightarrow 3\lambda - 12 > 0 \quad (\because \sin^2 x + \cos^2 x = 1)$$

$$\Rightarrow \lambda > 4$$

8. (B) : Putting $x^x = t$ & differentiating w.r.t. x , we get $x^x(1 + \log x)dx = dt$... (i)

$$\therefore \int \frac{x^x(x^{2x} + 1)(1 + \log x)}{x^{4x} + 1} dx$$

$$= \int \frac{x^x((x^x)^2 + 1)(1 + \log x)}{(x^x)^4 + 1} dx = \int \frac{t^2 + 1}{t^4 + 1} dt \quad [\text{Using (i)}]$$

$$= \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt = \int \frac{d\left(t - \frac{1}{t}\right)}{\left(t - \frac{1}{t}\right)^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t^2 - 1}{t\sqrt{2}} \right) + C$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^{2x} - 1}{x^x \sqrt{2}} \right) + C$$

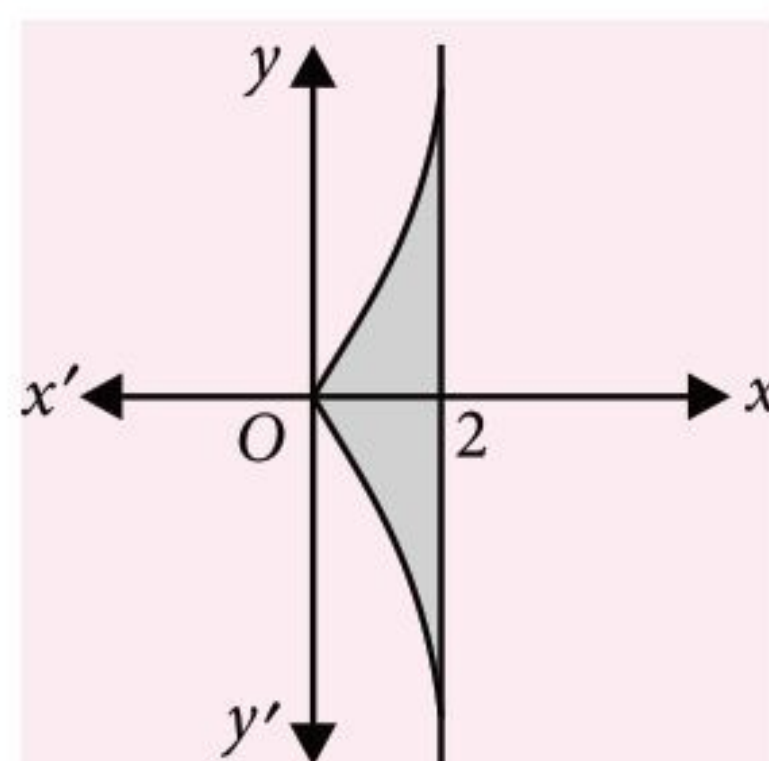
9. (C) : $|y| = x^3$

Clearly, $x \geq 0$

$$\therefore y = \pm x^3$$

Required area,

$$A = 2 \int_0^2 x^3 dx = 8 \text{ sq. units}$$



10. (A) : We have, $xe^{y/x} - y + x \frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y - xe^{y/x}}{x}$$

This is a homogeneous differential equation

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{vx - xe^v}{x} = v - e^v$$

$$\Rightarrow x \frac{dv}{dx} = -e^v \Rightarrow \int -\frac{1}{e^v} dv = \int \frac{dx}{x}$$

$$\Rightarrow e^{-v} = \log |x| + c \Rightarrow e^{-y/x} = \log |x| + c$$

It is given that $y(e) = 0 \Rightarrow 1 = (\log(e)) + c \Rightarrow c = 0$

$$\Rightarrow e^{-y/x} = \log |x| \Rightarrow \frac{-y}{x} = \log(\log |x|)$$

$\Rightarrow y = -x \log(\log |x|)$, $x \in (-\infty, -1) \cup (1, \infty)$, which is the required solution.

11. (D) : $|\vec{a} + \vec{b}| = 6 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 36$... (i)

Similarly, $|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = 64$... (ii)

and $|\vec{c}|^2 + |\vec{a}|^2 + 2\vec{c} \cdot \vec{a} = 100$... (iii)

Adding (i), (ii) and (iii), we get

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 100$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 100$$
 ... (iv)

$$(\because \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0)$$

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 100 \quad [\text{Using (iv)}]$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 10$$



**START THE JOURNEY TOWARDS
IMO OLYMPIAD SUCCESS NOW!**

Download Previous 5 Years' Papers!

CLASS
12



Scan & Download



DOWNLOAD NOW

12. (C)

13. (A) : Let E_1 be the event that letter is from TATA NAGAR and E_2 be the event that letter is from CALCUTTA.

Also, let E be the event that on the letter, two consecutive letters TA are visible.

$$\text{Therefore, } P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$$

If letter is from TATA NAGAR, then we choose two consecutive letters from set {TA, AT, TA, AN, NA, AG, GA, AR}.

So, $P(E|E_1) = \frac{2}{8}$ and if letter is from CALCUTTA, then

two consecutive letters can be chosen from {CA, AL, LC, CU, UT, TT, TA}. So, $P(E|E_2) = \frac{1}{7}$

$$\text{Hence, } P(E_1|E) = \frac{P(E_1) \cdot P(E|E_1)}{P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2)}$$

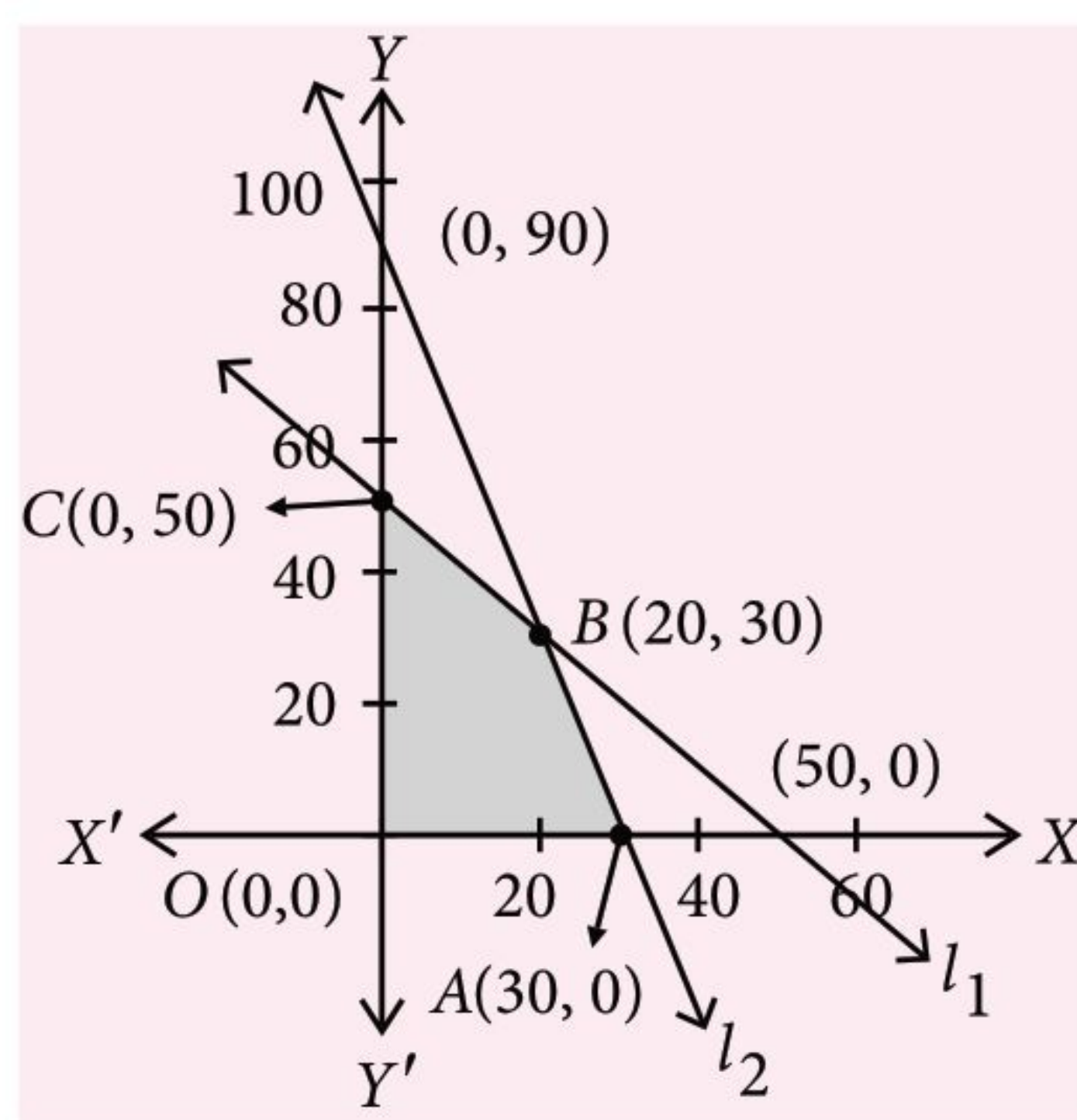
$$= \frac{\frac{1}{2} \cdot \frac{2}{8}}{\frac{1}{2} \cdot \frac{2}{8} + \frac{1}{2} \cdot \frac{1}{7}} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{1}{14}} = \frac{\frac{1}{8}}{\frac{11}{56}} = \frac{7}{11}$$

14. (C) : Let $l_1 : x + y = 50$

$l_2 : 3x + y = 90$

...(i)

...(ii)



From graph the corner points are $C(0, 50)$, $B(20, 30)$, $O(0, 0)$ and $A(30, 0)$.

The value of objective function at these corner points are as follows :

Corner Points	Value of $Z = 4x + y$
$O(0, 0)$	$Z = 0$
$A(30, 0)$	$Z = 120$ (Maximum)
$B(20, 30)$	$Z = 110$
$C(0, 50)$	$Z = 50$

$\therefore$ Maximum value of $Z = 120$ which is attained at point $(30, 0)$.

$$15. (C) : \text{Let } I = \int_{1/3}^3 \frac{1}{x} \log_e \left(\frac{x+x^2-1}{x-x^2+1} \right) dx$$

$$\text{Putting } x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$$

$$\therefore I = - \int_3^{1/3} t \log_e \left(\frac{\frac{1}{t} + \frac{1}{t^2} - 1}{\frac{1}{t} - \frac{1}{t^2} + 1} \right) \frac{1}{t^2} dt$$

$$\Rightarrow I = \int_{1/3}^3 \frac{1}{t} \log_e \left(\frac{t-t^2+1}{t+t^2-1} \right) dt$$

$$\Rightarrow I = - \int_{1/3}^3 \frac{1}{x} \log_e \left(\frac{x+x^2-1}{x-x^2+1} \right) dx \Rightarrow I = -I \Rightarrow I = 0$$

$$16. (C) : (i) I = \int_0^{2a} \frac{f(x)}{f(x) + f(2a-x)} dx$$

$$\therefore I = \int_0^{2a} \frac{f(2a-x)}{f(2a-x) + f(x)} dx$$

$$\left(\text{Using } \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$2I = \int_0^{2a} dx$$

$$\therefore I = a$$

$$(ii) \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} \Rightarrow \Delta_1 = \Delta_2$$

where Δ_1 is the area of triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) and Δ_2 is the area of triangle with vertices (a_1, b_1) and (a_2, b_2) and (a_3, b_3) . But two Δ 's of same area may not be congruent.

$\therefore$ Given statement is false.

(iii) Given : $P(A) = 0.2$, $P(B) = 0.3$

$$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

(As A, B are independent events)

$$= 0.2 + 0.3 - (0.2)(0.3) = 0.44 \neq 0.5.$$

17. (B) : (i) With 0 and 1 as elements there are $2 \times 2 \times 2 \times 2 = 16$ determinants of order 2×2 out of which only

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \text{ are the three determinants}$$

whose value is +ve.

$\therefore$ Required probability = $3/16$

$$(ii) [x^2] = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < \sqrt{2} \\ 2 & \text{if } \sqrt{2} \leq x \leq 1.5 \end{cases}$$

$$\therefore \int_0^{1.5} [x^2] dx = \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{1.5} 2 dx$$

$$= (\sqrt{2} - 1) + 2(1.5 - \sqrt{2}) = \sqrt{2} - 1 + 3 - 2\sqrt{2} = 2 - \sqrt{2}$$

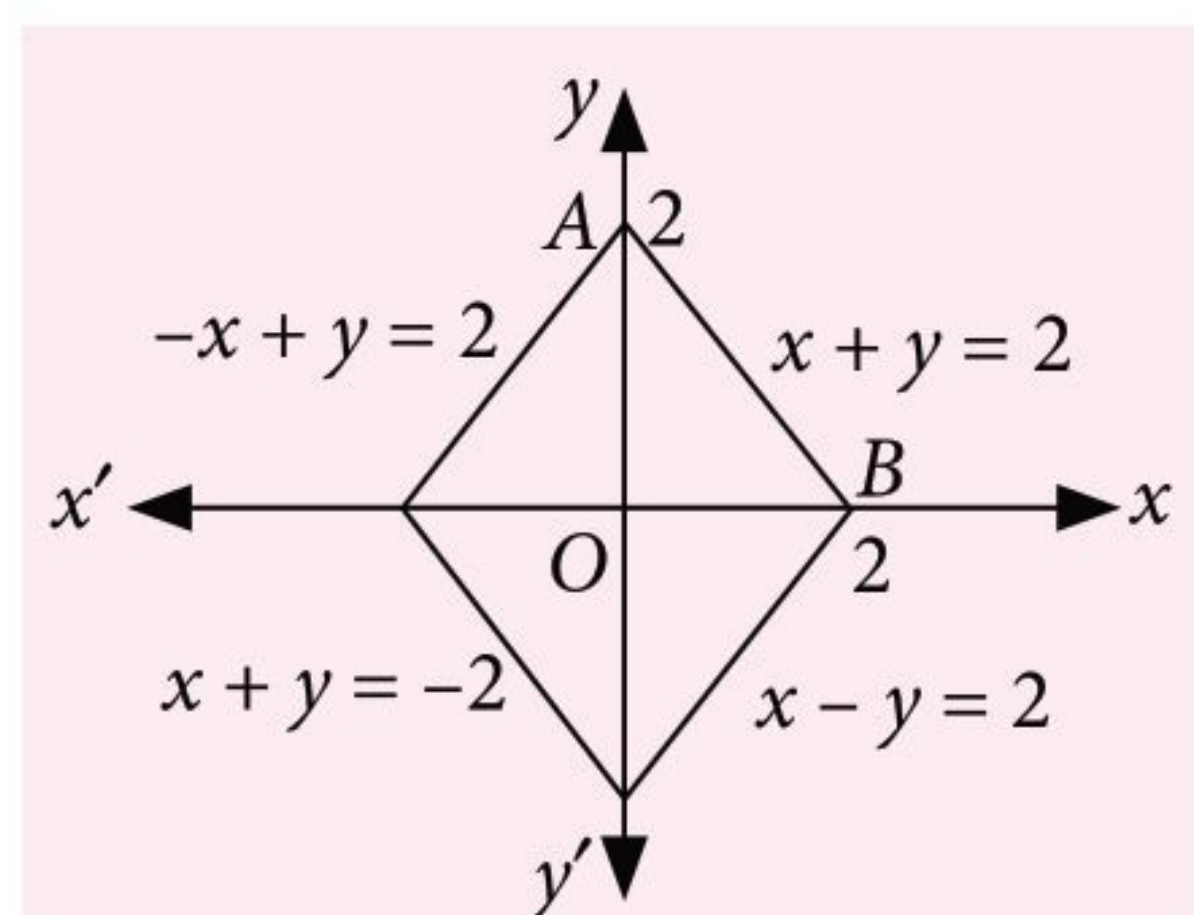
$$(iii) P(B/(A \cup B^C)) = \frac{P(B \cap (A \cup B^C))}{P(A \cup B^C)}$$

$$= \frac{P((B \cap A) \cup (B \cap B^C))}{P(A \cup B^C)}$$

$$= \frac{P(A \cap B)}{P(A \cup B^C)} = \frac{P(A) - P(A \cap B^C)}{P(A \cup B^C)}$$

$$= \frac{(1 - 0.3) - 0.5}{(1 - 0.3) + (1 - 0.4) - 0.5} = 0.25$$

18. (A) : We have, $|x| + |y| = 2$



$$AB = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\therefore \text{Required area} = (\sqrt{8})^2 = 8$$

19. (C) : (P) $f'(x) = 9(3x^2 - 4x + 1) = 0 \Rightarrow x = 1, \frac{1}{3}$

$$\therefore f = f\left(\frac{1}{3}\right) = 9 \cdot \frac{1}{3} \cdot \frac{4}{9} = \frac{4}{3}$$

$$f(0) = 0; f(1) = 0, f(2) = 18. \text{ Hence, } f_{\max}(2) = 18.$$

(Q) $f'(x) = \frac{4x}{(x^2 + 1)^2} = 0 \Rightarrow x = 0 \therefore f_{\min} = f(0) = -1$

(R) $f'(x) = -\frac{4x}{(x^2 - 1)^2} = 0 \Rightarrow x = 0 \therefore f_{\max} = f(0) = -1$

(S) $f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$

$$\therefore f_{\max} = f(-1) = -2 \text{ and } f_{\min} = f(1) = 2$$

20. (A) : We have, $\frac{x \frac{dy}{dx} - y}{\sqrt{x^2 - y^2}} = 10x^2$

$$\Rightarrow \frac{x dy - y dx}{x^2} = 10x \cdot \sqrt{1 - \frac{y^2}{x^2}} dx \quad \dots(i)$$

Put $\frac{y}{x} = t \Rightarrow \frac{x dy - y dx}{x^2} = dt$

$$\therefore (i) \text{ becomes, } \frac{dt}{\sqrt{1-t^2}} = 10x dx$$

Integrating both sides, we get $\int \frac{dt}{\sqrt{1-t^2}} = 10 \int x dx$

$$\Rightarrow \sin^{-1} t = \frac{10x^2}{2} + C \Rightarrow \sin^{-1} \left(\frac{y}{x} \right) - 5x^2 = C$$

For other sections/subjects please refer to
Physics For You, Chemistry Today and Biology Today



mtg

JEE ADVANCED EXPLORER

Dry runs are here!

FEATURES

- 10 years solved papers with detailed solutions
- 10 Model Test Papers with Detailed Solutions
- Chapterwise indexing of questions

Visit www.mtg.in
to buy online!



Challenging PROBLEMS For JEE

ON Application of Derivatives



MULTIPLE CORRECT ANSWER TYPE

1. Let $f(x) = \frac{x-1}{x^2}$ then which of the following is/are correct?

- (a) $f(x)$ has minima but no maxima.
- (b) $f(x)$ increases in the interval $(0, 2)$ and decreases in the interval $(-\infty, 0) \cup (2, \infty)$.
- (c) $f(x)$ is concave downward in $(-\infty, 0) \cup (0, 3)$.
- (d) $x = 3$ is the point of inflection.

2. If $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$, $a > 0$, $0 \leq x \leq 2a$, then which of the following is/are correct?

- (a) $\phi(x)$ increases in $(a, 2a)$
- (b) $\phi(x)$ increases in $(0, a)$
- (c) $\phi(x)$ decreases in $(0, a)$
- (d) $\phi(x)$ decreases in $(a, 2a)$

3. If the function $f(x) = (\sqrt{4-x^2} - 3)^2 + (\sqrt{4-x^2} + 1)^3$, then which of the following is/are correct? ($[\cdot]$ denotes GIF).

- (a) $f(x)$ has greatest value 28
- (b) $f(x)$ has least value $K \Rightarrow [K] = 9$
- (c) The slope of tangent of $f(x)$ at $x = 1$ equals $-(8 + 2\sqrt{3})$
- (d) $f'(2) = 0$

4. If a is a negative integer such that at each point of the curve $f(x) = y = x^3 - ax^2 + x + 1$ the tangent is having positive slope, then which of the following is/are correct?

- (a) $f(x) = 0$ has a negative real root
- (b) $f(x) > 0 \forall x > 0$
- (c) $f(x)$ has no point of inflection
- (d) $f(x)$ is concave upward in $(0, \infty)$

5. Let $f: R \rightarrow R$ be defined as $f(x) = \sin x + ax + b$ ($a, b \in R$). Then the equation $f(x) = 0$ has

- (a) only one real root which is positive if $a > 1$, $b < 0$
- (b) only one real root which is negative if $a > 1$, $b > 0$
- (c) only one real root which is negative if $a < -1$, $b < 0$
- (d) only one real root which is positive if $a < -1$, $b < 0$

6. Let $f(x) = \begin{cases} 0 & , x = 0 \\ (1-x)^\alpha x^\beta (1 - \cos 2\pi x), & 0 < x < 1 \\ 0 & , x = 1 \end{cases}$. If

Rolle's theorem is applicable to $f(x)$ for $x \in [0, 1]$, then

- (a) $\alpha \in R$ (b) $\beta \in R$
- (c) $\beta \in (-2, \infty)$ (d) $\alpha \in (-2, \infty)$

7. Let $f(x)$ be a twice differentiable function such that $f''(x) > 0$ in $[0, 2]$. Then

- (a) $f(0) + f(2) = 2f(c)$ for atleast one $c \in (0, 2)$
- (b) $f(0) + f(2) < 2f(1)$
- (c) $f(0) + f(2) > 2f(1)$
- (d) $2f(0) + f(2) > 3f\left(\frac{2}{3}\right)$

8. Let $f(x) = x^2 e^{-x^2}$, then

- (a) $f(x)$ has local maxima at $x = -1$ and $x = 1$
- (b) $f(x)$ has local minima at $x = 0$
- (c) $f(x)$ is strictly decreasing on $x \in R$
- (d) Range of $f(x)$ is $\left[0, \frac{1}{e}\right]$

9. Let $f(x) = \begin{cases} 2x - 4 & ; x \leq 2 \\ -x^2 + \frac{k^3(k-1)^2}{k^2 - k - 2} + 4 & ; x > 2 \end{cases}$

$f(x)$ attains local maximum at $x = 2$, if k lies in

- (a) $(0, 1)$ (b) $(3, \infty)$
- (c) $(-\infty, -1)$ (d) $(1, 2)$

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

10. Consider $f(x) = \begin{cases} x^3 - 3x + 3 & ; x \in [-2, 0) \\ x + 1 & ; x \in [0, 5] \\ 1 - e^{-x} & ; x > 5 \end{cases}$, then

- (a) $f(x)$ has local maxima at two points
 (b) $f(x)$ has no local minima
 (c) Global minima of $f(x)$ does not exist
 (d) Global maxima value of $f(x)$ is 6

11. The extremum value of the function

$$f(x) = \frac{1}{\sin x + 4} - \frac{1}{\cos x - 4}, \quad x \in R, \text{ is}$$

- (a) $\frac{4}{8 - \sqrt{2}}$ (b) $\frac{2\sqrt{2}}{8 - \sqrt{2}}$
 (c) $\frac{2\sqrt{2}}{4\sqrt{2} + 1}$ (d) $\frac{4\sqrt{2}}{8 + \sqrt{2}}$

12. Which of the following is true?

- (a) The least natural number 'a' for which $x + ax^{-2} > 2$, $\forall x \in (0, \infty)$ is 2.
 (b) The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is 2
 (c) If $a, b \in R$ distinct numbers satisfying, $|a - 1| + |b - 1| = |a| + |b| = |a + 1| + |b + 1|$, then the minimum value of $|a - b|$ is 2.
 (d) None of these

13. The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at $x =$

- (a) -2 (b) $-\frac{2}{3}$ (c) 2 (d) $\frac{2}{3}$

14. Let $f(x) = (x - 1)^4 (x - 2)^n$; $n \in N$, then $f(x)$ has

- (a) a maximum at $x = 1$, if n is odd
 (b) a maximum at $x = 1$, if n is even
 (c) a minimum at $x = 1$, if n is even
 (d) a minima at $x = 2$, if n is even

15. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S . If a, b, c and d denote the lengths of the sides of the quadrilateral.

- (a) minimum value of $a^2 + b^2 + c^2 + d^2 = 2$
 (b) maximum value of $a^2 + b^2 + c^2 + d^2 = 4$
 (c) minimum value of $a^2 + b^2 + c^2 + d^2 = 1$
 (d) maximum value of $a^2 + b^2 + c^2 + d^2 = 3$

16. If $|f(x)| = |ax^2 + bx + c| \leq 1$ for $|x| \leq 1$ ($a, b, c \in R$) and $\frac{8}{3}a^2 + 2b^2$ is maximum then

- (a) $b = 0$ (b) $|a + c| = 1$
 (c) $ac = 1$ (d) $\left|\frac{c}{a}\right| = \frac{1}{2}$

17. Let $f(x) = x^3 + ax^2 + bx + c$ where $a, b, c \in R$, then which of the following statement(s) is (are) correct?

- (a) If the equation $f(x) = 0$ has exactly one real root, then $f(x)$ must be strictly increasing on R .
 (b) If $f(x)$ has a negative point of local minimum, then both roots of equation $f'(x) = 0$ must be negative and distinct.
 (c) If $f(x_1) \cdot f(x_2) < 0$, $x_1 < x_2$, then the equation $f(x) = 0$ has atleast one real root in (x_1, x_2) .
 (d) $f(x)$ possesses exactly one point of inflexion.

18. If $f(x) = \frac{x^5}{5} + \frac{x^4}{4} + x^3 + \frac{kx^2}{2} + x$ be a real valued function. Then greatest value of k^2 for which $f(x)$ is increasing function $\forall x \in R$ is less than or equal to

- (a) 12 (b) 11 (c) 10 (d) 9

19. Let $f_N(x)$ be a polynomial function which satisfies

$$f'_N(x) = \prod_{n=1}^N (x - n)^n. \text{ If } n \text{ is number of values of } x \text{ for which } f_{2018}(x) \text{ attains local maxima and } n_2 \text{ is number of values of } x \text{ for which } f_{2018}(x) \text{ attains local minima. Then}$$

(a) $n_2 = 505$ (b) $n_1 = 504$
 (c) $n_2 = 1009$ (d) $n_1 = 1009$

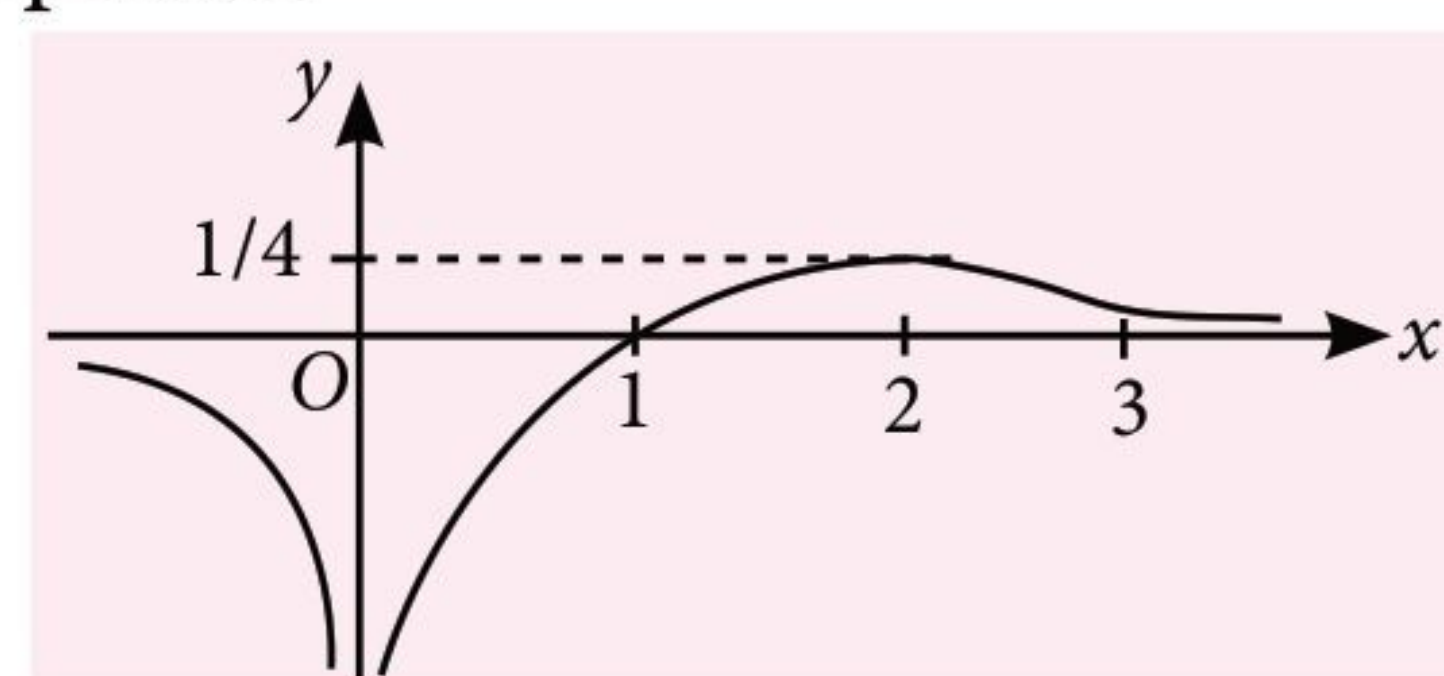
20. The function $f(x) = x^{1/3} (x - 1)$

- (a) has 2 inflection points.
 (b) is strictly increasing for $x > 1/4$ and strictly decreasing for $x < 1/4$.
 (c) is concave downward in $(-1/2, 0)$.
 (d) area enclosed by the curve and x axis lying in the fourth quadrant is $9/28$.

SOLUTIONS

1. (b, c, d): $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{2(x-3)}{x^4}$.

Now interpretate



2. (a, c): $\phi(x) = f(x) + f(2a - x)$

$$\phi'(x) = f'(x) - f'(2a - x)$$

Since $f''(x) > 0 \Rightarrow f'(x)$ is an increasing function

Hence $f'(x) > f'(2a - x) \Rightarrow x > 2a - x \Rightarrow x > a$

$\therefore \phi'(x) > 0$ if $x > a \Rightarrow \phi(x)$ is increasing in $(a, 2a)$

Similarly, $\phi(x)$ is decreasing in $(0, a)$.

3. (a, b, c): $x \in [-2, 2] \Rightarrow a \in [0, 2]$, where $a = \sqrt{4 - x^2}$
 $\phi(a) = (a - 3)^2 + (a + 1)^3$
 $\phi'(a) = 3a^2 + 8a - 3 = (3a - 1)(a + 3)$
 $\therefore \phi_{\min} = \phi(1/3) = \frac{64}{9} + \frac{64}{27} = \frac{256}{27}$

$\phi_{\max} = \phi(2) = 28$
 $\frac{dy}{dx} = (6 + 8\sqrt{3}) \frac{1}{2\sqrt{4-1}} (-2 \times 1) \quad (x=1 \Rightarrow a=\sqrt{3})$
 $= -\left(\frac{6+8\sqrt{3}}{\sqrt{3}}\right) = -(8+2\sqrt{3})$

4. (a, b, d): $f'(x) = 3x^2 - 2ax + 1 > 0 \forall x \in \mathbb{R}$
 $\Rightarrow a \in (-\sqrt{3}, \sqrt{3}) \Rightarrow a = -1$
 $\Rightarrow f(x) = x^3 + x^2 + x + 1 = (x^2 + 1)(x + 1)$

5. (a, b, c): $f: \mathbb{R} \rightarrow \mathbb{R}$

and $f(x) = \sin x + ax + b \Rightarrow f'(x) = \cos x + a$

If $a > 1$, then f is increasing

If $a < -1$, then f is decreasing

If $a > 1$, $f(0) = b$ and $b < 0 \Rightarrow f(0) < 0$

as shown in the diagram

$\therefore f(x) = 0$ has only the +ve root

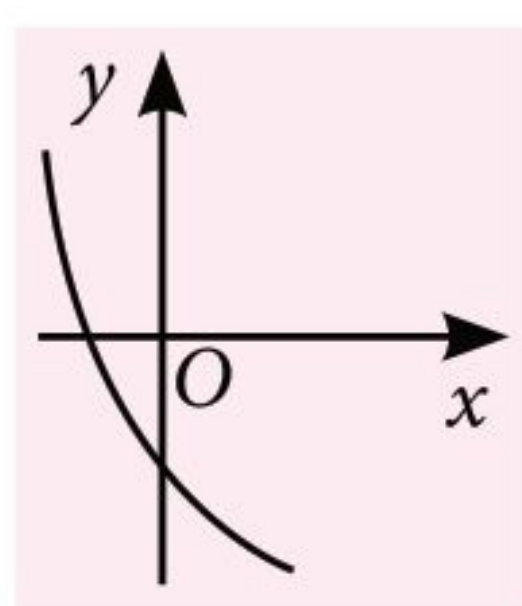
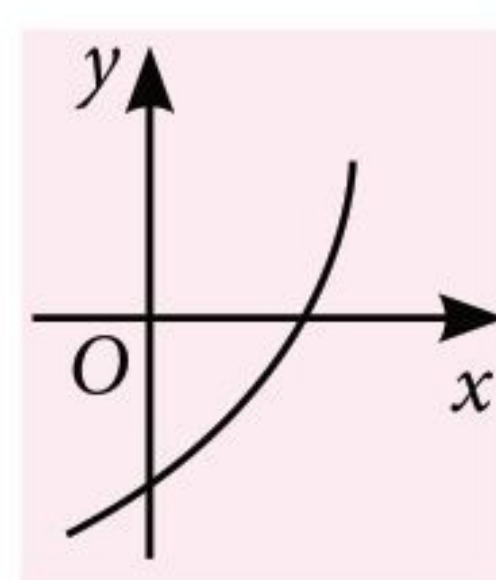
hence option (a) is correct

If $f(0) > 0$ again f has only one

negative root

If $a < -1$, $b < 0$, then f is decreasing

and has one negative real root.



6. (c, d): Since, $f(0) = 0, f(1) = 0$

$f(x)$ is continuous at $x = 0 \Rightarrow f(0^+) = f(0)$

$\Rightarrow 0 = \lim_{x \rightarrow 0^+} 2(1-x)^\alpha x^\beta (\sin^2 \pi x)$

$= \lim_{x \rightarrow 0^+} 2\pi^2 (1-x)^\alpha x^2 \cdot x^\beta = \lim_{x \rightarrow 0^+} 2\pi^2 (1-x)^\alpha x^{\beta+2},$

when $\beta + 2 > 0 \Rightarrow \beta > -2$

But for continuity at $x = 1$

$f(1) = f(1^-) \Rightarrow 0 = \lim_{h \rightarrow 0^+} h^\alpha (1-h)^\beta (1 - \cos(2\pi - 2\pi h))$

$\Rightarrow 0 = \lim_{h \rightarrow 0^+} h^\alpha (1-h)^\beta (2 \sin^2 \pi h)$

$\Rightarrow \lim_{h \rightarrow 0^+} h^\alpha (1-h)^\beta \frac{2(\sin^2 \pi h)}{(\pi h)^2} (\pi h)^2 = 0$

$\Rightarrow 2\pi^2 \lim_{h \rightarrow 0^+} h^{\alpha+2} = 0 \Rightarrow \alpha + 2 > 0 \Rightarrow \alpha > -2$

$f(x)$ is derivable in $(0, 1)$

Therefore Rolle's theorem is applicable if $\alpha > -2, \beta > -2$.

7. (a, c, d): $X \in [0, 2]$ and $f''(x) > 0$

$\Rightarrow f'(x)$ is increasing in $[0, 2] \Rightarrow$ LMVT is applicable

Let $c_1 \in (0, 1)$ and $c_2 \in (1, 2)$

By LMVT, $f'(c_1) = \frac{f(1) - f(0)}{1 - 0}$

$f'(c_2) = \frac{f(2) - f(1)}{2 - 1}$

$f'(x)$ is increasing and $c_1 < c_2$

$\Rightarrow f'(c_1) < f'(c_2) \Rightarrow f(1) - f(0) < f(2) - f(1)$

$f(0) + f(2) > 2f(1)$

Let $c_1 \in (0, 2/3)$ and $c_2 \in (2/3, 2)$; $f'(c_1) < f'(c_2)$

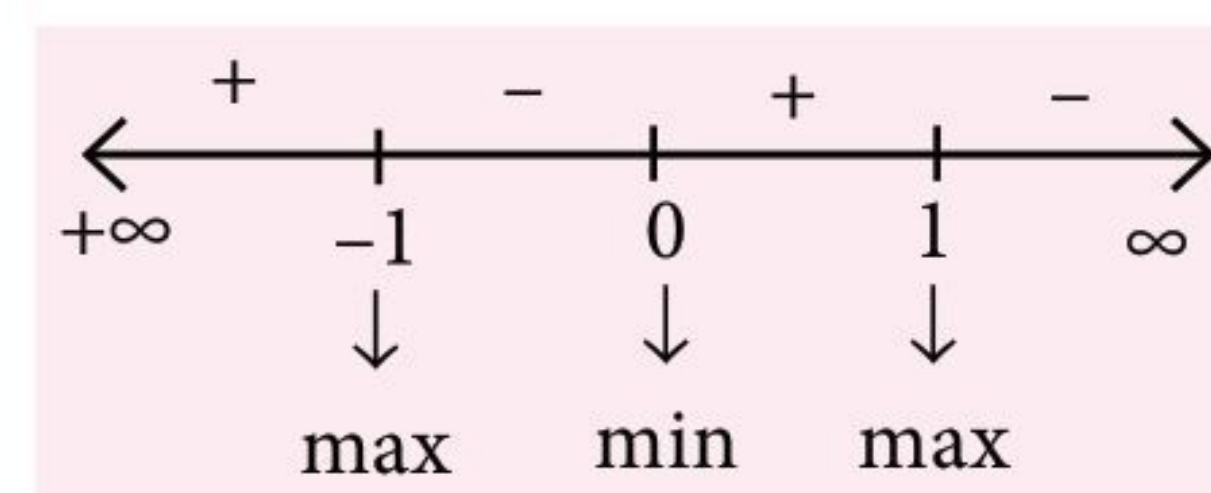
$\Rightarrow \frac{f(2/3) - f(0)}{\frac{2}{3} - 0} < \frac{f(2) - f(2/3)}{2 - 2/3}$

$\Rightarrow 2f(0) + f(2) > 3f(2/3)$

8. (a, b, d): $f(x) = x^2 e^{-x^2}$

$f'(x) = 2xe^{-x^2} (1 - x^2)$

$f(x)$ has local maximum at $x = -1$ and 1 and local minimum at $x = 0$.



$f(0) = 0, f(1) = \frac{1}{e}, \text{ As } x \rightarrow \infty \Rightarrow f(x) \rightarrow 0$

So, Range of $f(x)$ is $\left[0, \frac{1}{e}\right]$.

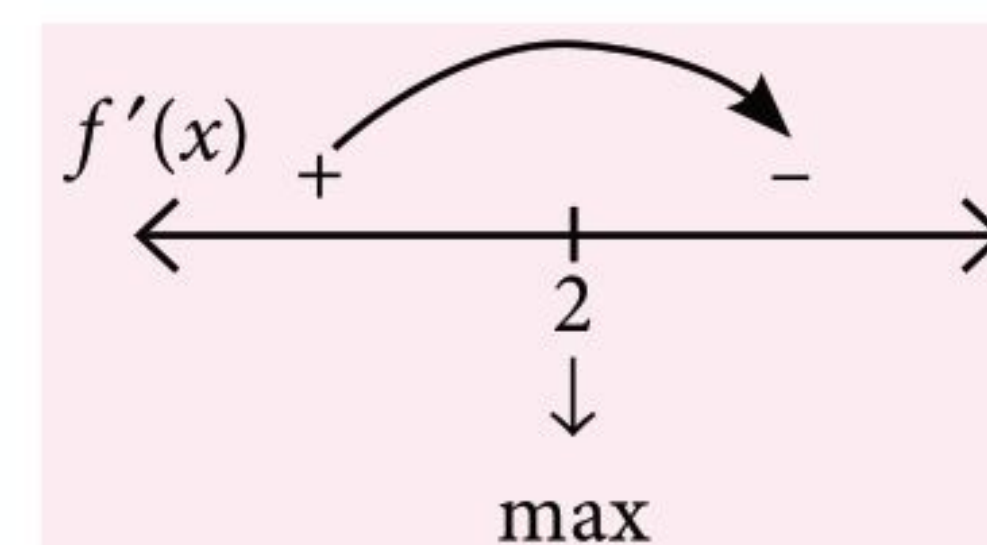
9. (a, c, d): When $f(x)$ is continuous at $x = 2$

$f(2^-) = f(2^+) \Rightarrow 0 = \frac{k^3(k-1)^2}{k^2 - k - 2} \Rightarrow k = 0, 1$

When $f(x)$ is discontinuous

at $x = 2$

$f'(x) = \begin{cases} 2 & ; x \leq 2 \\ -2x & ; x > 2 \end{cases}$

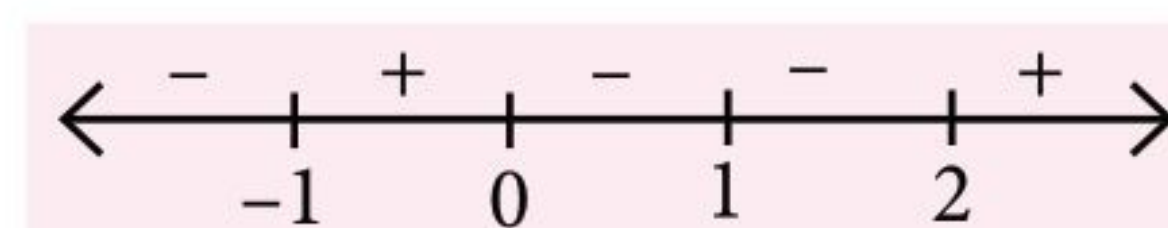


$f'(x)$ does not exist at $x = 2$, and $f'(x)$ changes from +ve to -ve

$\Rightarrow f(x)$ attains maximum at $x = 2$

$\Rightarrow \lim_{x \rightarrow 2^-} f(x) = f(2) > \lim_{x \rightarrow 2^+} f(x)$

$\Rightarrow 0 > \frac{k^3(k-1)^2}{(k-2)(k+1)}$



$\Rightarrow k^3(k-1)^2(k-2)(k+1) < 0$ and $k \neq 2, -1$

$\therefore k \in (-\infty, -1) \cup [0, 2)$

10. (a, c, d): $f(x)$ is continuous $[-2, 0]$

$\Rightarrow f(x)$ has local maximum, at $x = -1$,

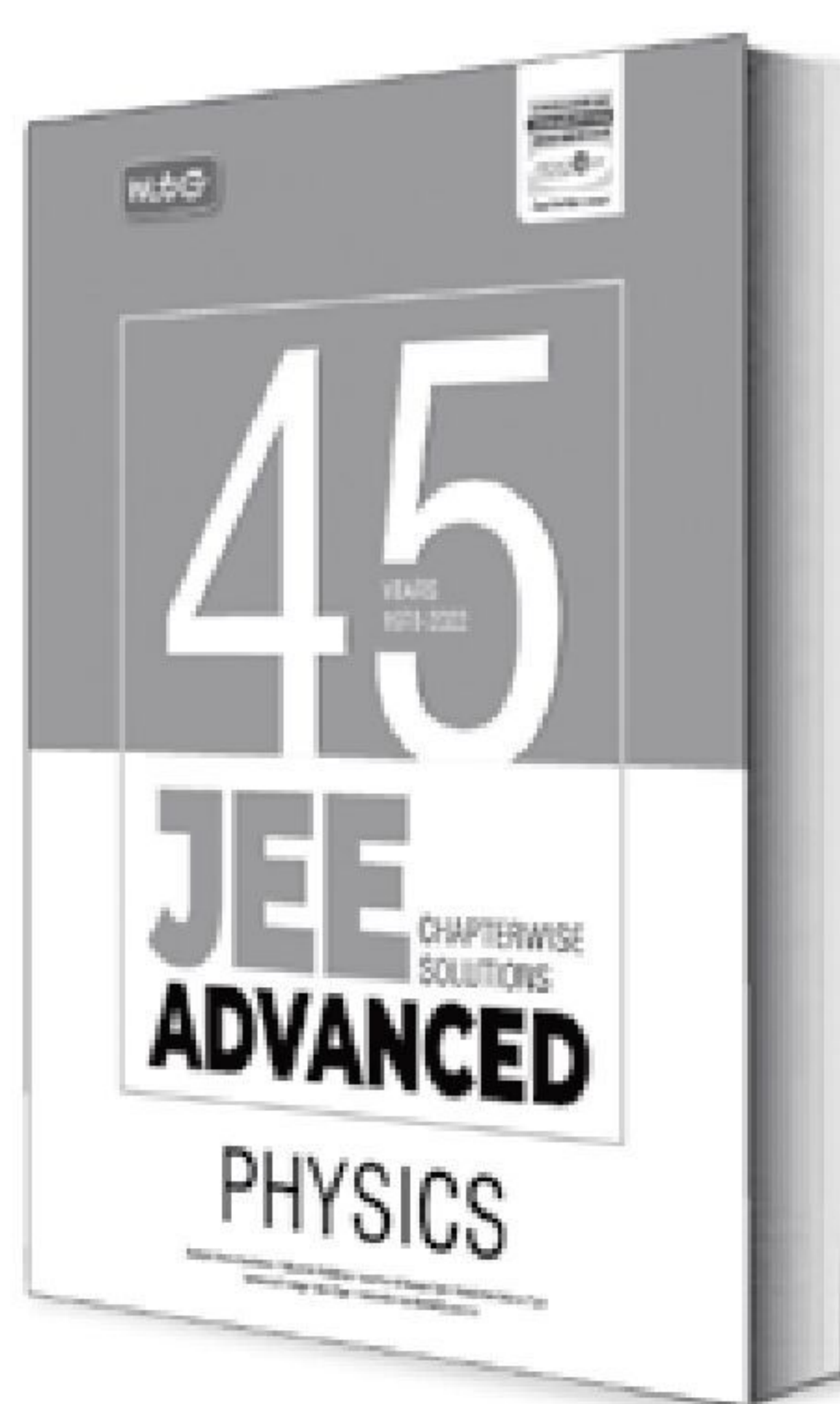
and discontinuous at $x = 5$

But $f(x)$ has local maximum at $x = 5$

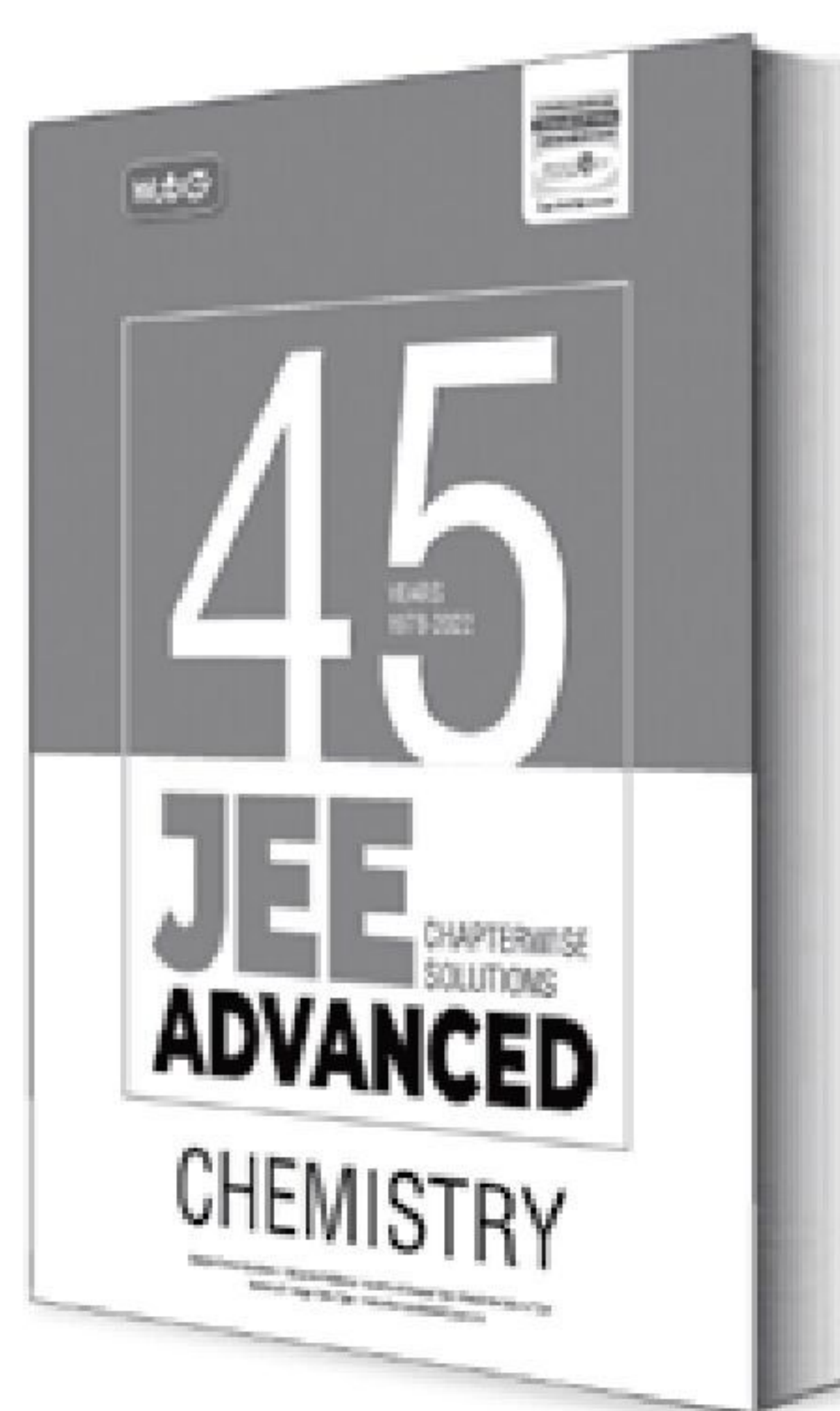
$(f(5) > f(5^+))$



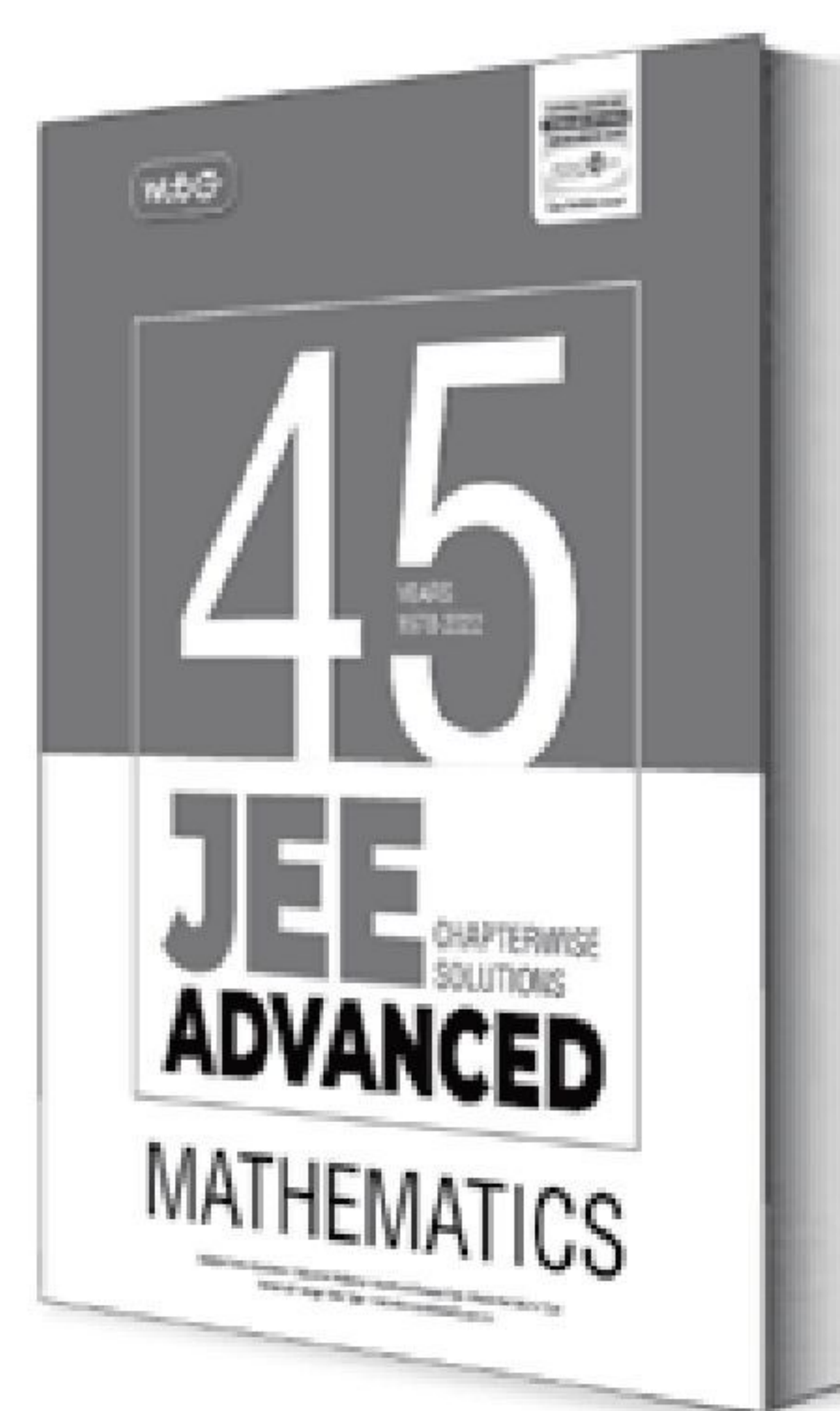
How can history help to succeed in JEE!



₹625/-



₹625/-



₹625/-

Wouldn't you agree that previous years' test papers provide great insights into the pattern and structure of future tests. Studies corroborate this, and have shown that successful JEE aspirants begin by familiarising themselves with problems that have appeared in past JEEs, as early as 2 years in advance.

Which is why the MTG team created 45 Years Chapterwise Solutions. The most comprehensive 'real' question bank out there, complete with detailed solutions by experts. An invaluable aid in your quest for success in JEE. Visit www.mtg.in to order online. Or simply scan the QR code to check for current offers.



Scan now with your
smartphone or tablet

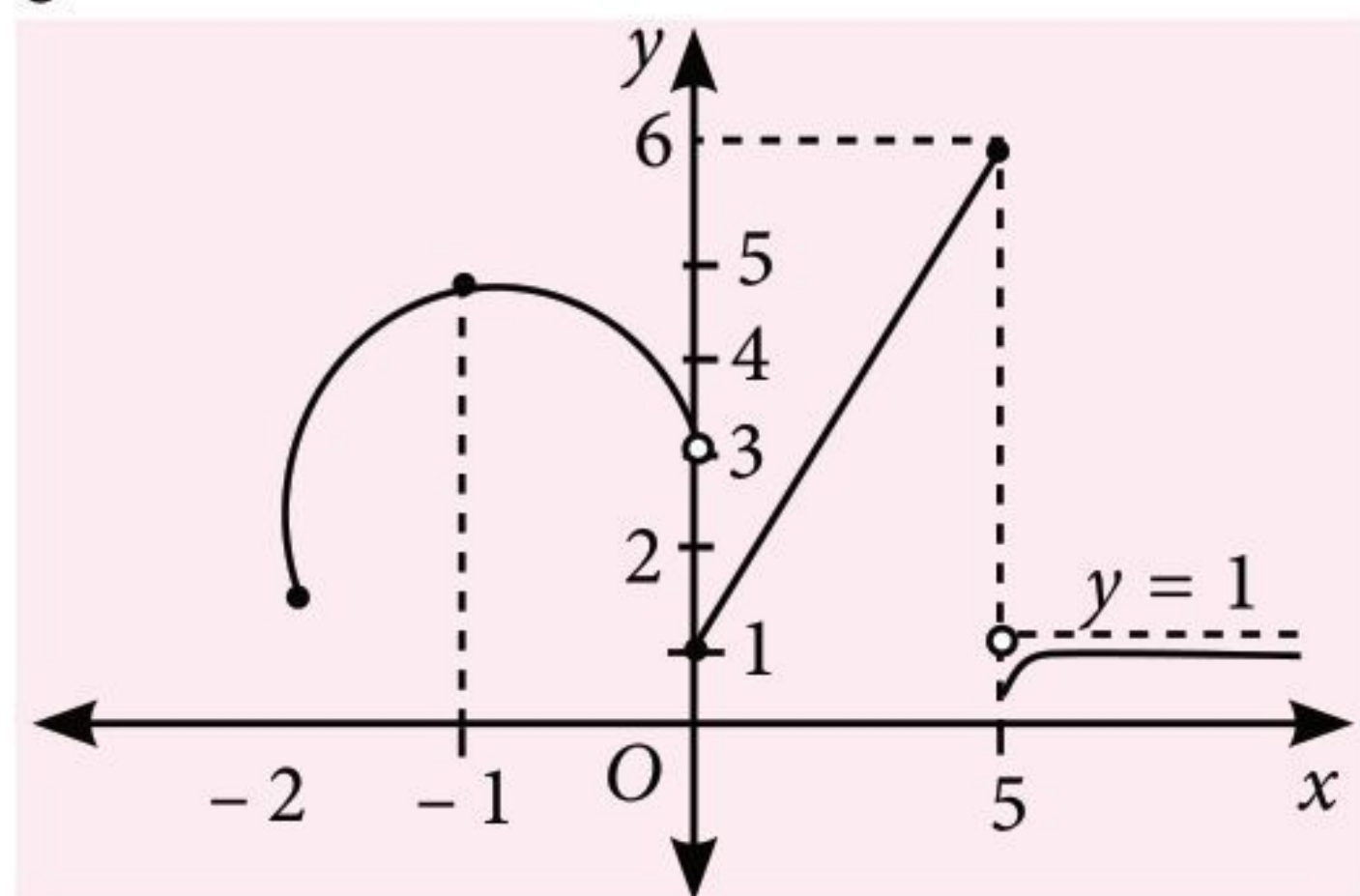
Application to read
QR codes required

Note: 45 Years Chapterwise Solutions are also available for each subject separately.

Available at all leading book shops throughout India. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or e-mail info@mtg.in

Global minima of $f(x)$ does not exist and global maxima = 6



11. (a, c): $f(x) = \frac{1}{\sin x + 4} - \frac{1}{\cos x - 4}$
 $\Rightarrow f'(x) = \frac{-\cos x}{(\sin x + 4)^2} - \frac{\sin x}{(\cos x - 4)^2}$
 $\Rightarrow f'(x) = 0 \Rightarrow (\sin x + \cos x)[1 - \sin x \cos x + 16 + 8(\sin x - \cos x)] = 0$
 $\Rightarrow \sin x + \cos x = 0 \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$

Global minimum: $x = 2n\pi + \left(\frac{3\pi}{4}\right)$

Global maximum: $x = 2n\pi + \left(\frac{7\pi}{4}\right)$

$M = \frac{4}{8 - \sqrt{2}}, m = \frac{4}{8 + \sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2} + 1}$

12. (a, c): (a) $x + \frac{a}{x^2} > 2, \forall x \in (0, \infty)$
 $\Rightarrow \frac{x}{2} + \frac{x}{2} + \frac{a}{x^2} > 2$. Now, $\frac{\frac{x}{2} + \frac{x}{2} + \frac{a}{x^2}}{3} \geq \left(\frac{x}{2} \cdot \frac{x}{2} \cdot \frac{a}{x^2}\right)^{1/3}$
 $(\because \text{AM} \geq \text{GM})$

$\Rightarrow x + ax^{-2} \geq 3 \frac{a^{1/3}}{4^{1/3}} > 2 \Rightarrow \frac{3^3 a}{4} > 8 \Rightarrow a > \frac{32}{27}$

(b) $f(x, y, z) = x^2 - 2x + 1 + 4y^2 - 12y + 9 + 3z^2 - 6z + 3 + 1 = (x - 1)^2 + (2y - 3)^2 + 3(z - 1)^2 + 1$

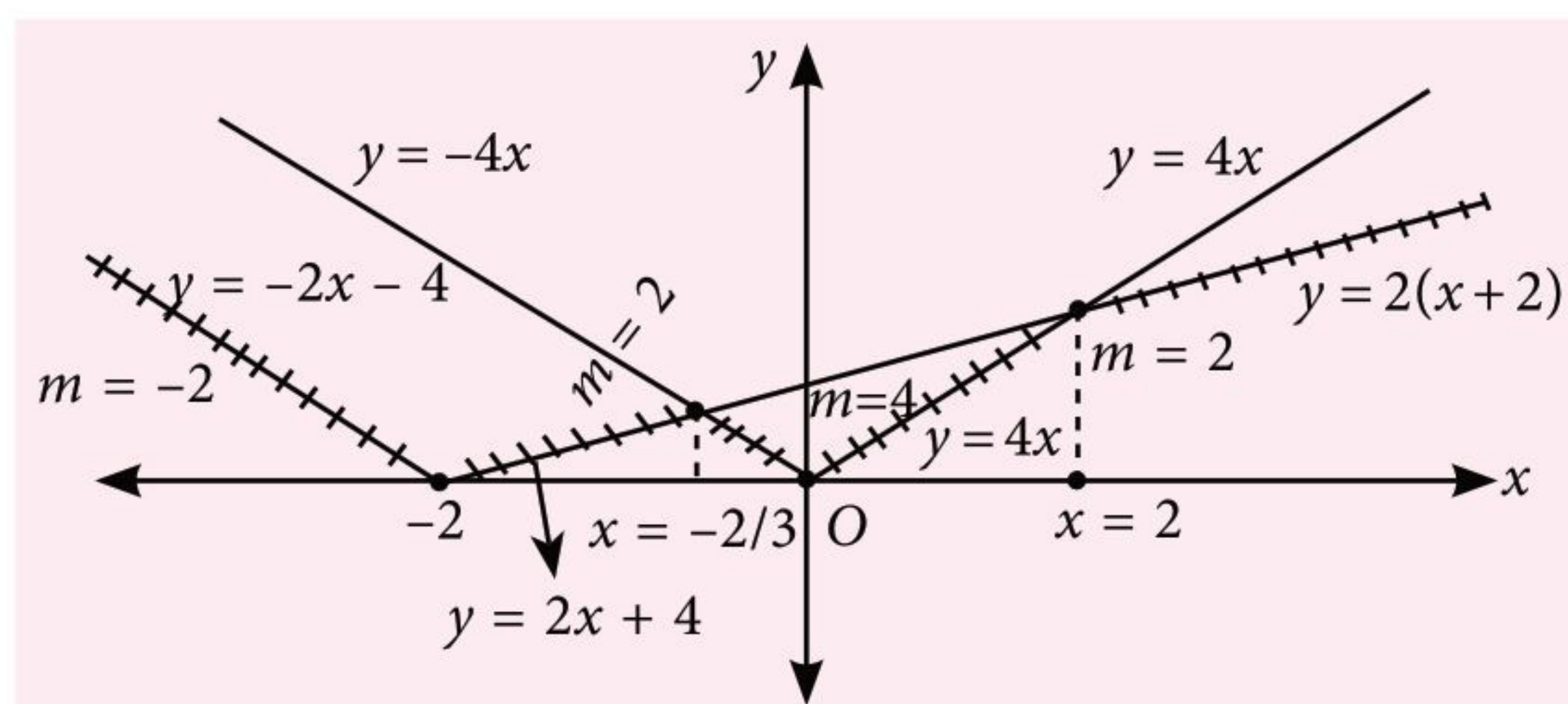
Hence least value of $f(x, y, z) = 1$, when $x = 1$,

$y = \frac{3}{2}, z = 1$

(c) Let $a < b$ and $f(x) = |x - a| + |x - b| \forall x \in R$
 So, $f(x)$ is decreasing in $(-\infty, a]$, constant in $[a, b]$ and increasing in $[b, \infty)$. We have, $f(0) = f(1) = f(-1)$
 $\Rightarrow \{-1, 0, 1\} \in [a, b] \therefore |a - b|_{\min} = 2$

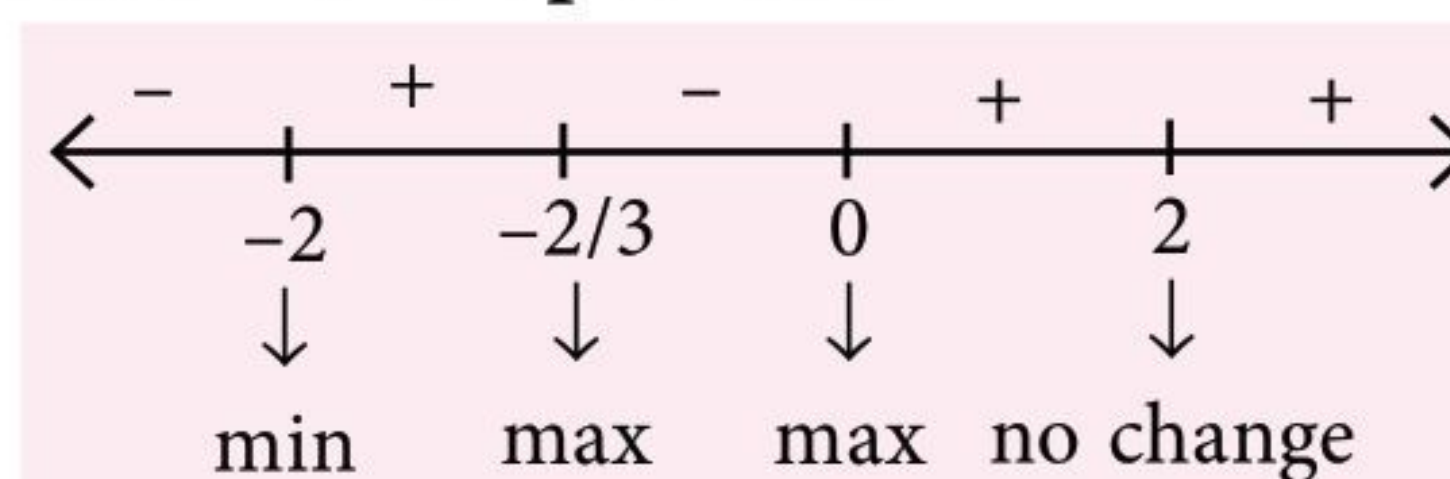
13. (a, b): $\frac{f + g - |f - g|}{2}$
 $= \begin{cases} \frac{f + g - (f - g)}{2} = g(x) = \min(f, g) & (\text{if } f > g) \\ \frac{f + g + f - g}{2} = f(x) = \min(f, g) & (\text{if } f < g) \end{cases}$

$\left[\frac{f + g - |f - g|}{2} = \min(f(x), g(x)) \right]$
 $\Rightarrow \frac{2|x| + |x + 2| - ||x + 2| - 2|x||}{2} = \min(|x + 2|, 2|x|)$



$y = -4x; y = 2x + 4 \Rightarrow 2x + 4 = -4x \Rightarrow 6x = -4 \Rightarrow x = -2/3$
 $4x = 2x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$

Number of points of local maximum/minimum = 3 (by observation of slopes 'm')

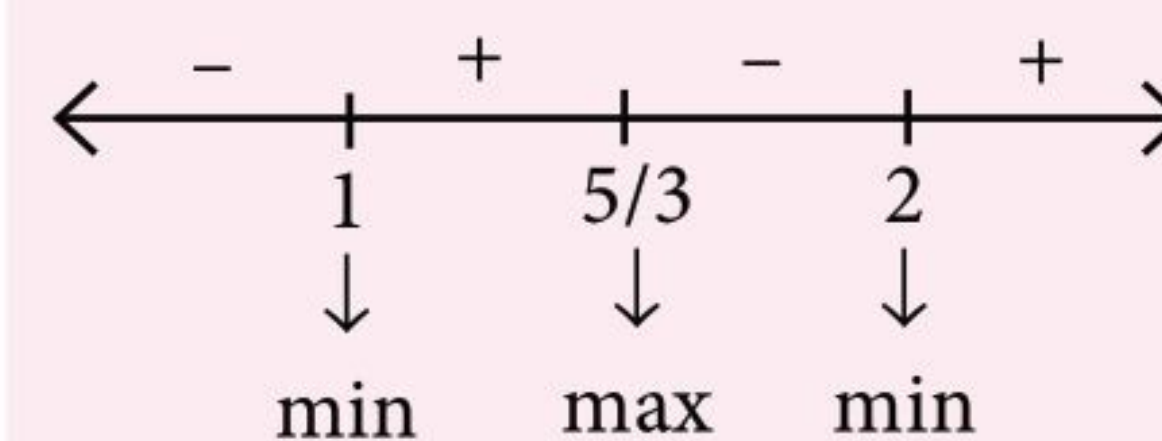
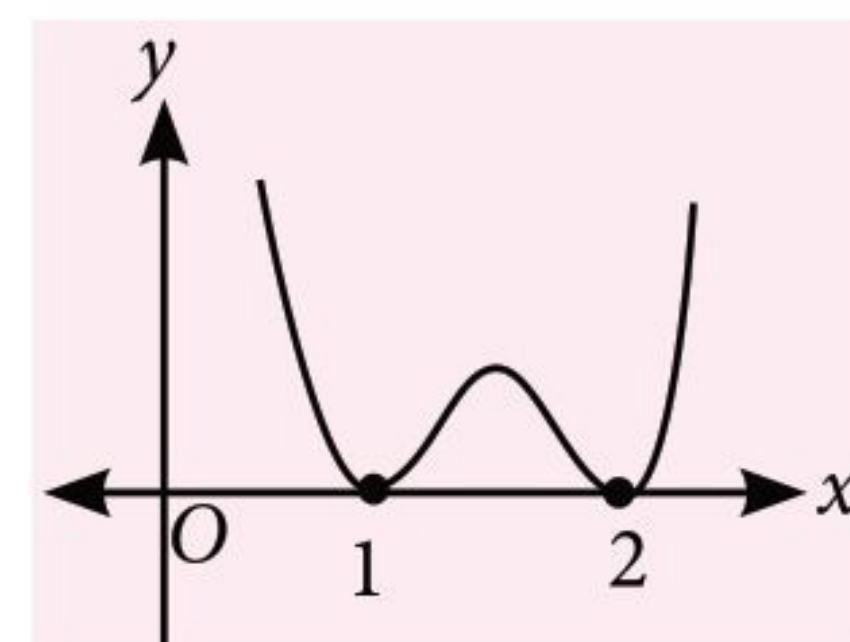


14. (a, c, d): Graph of $f(x)$

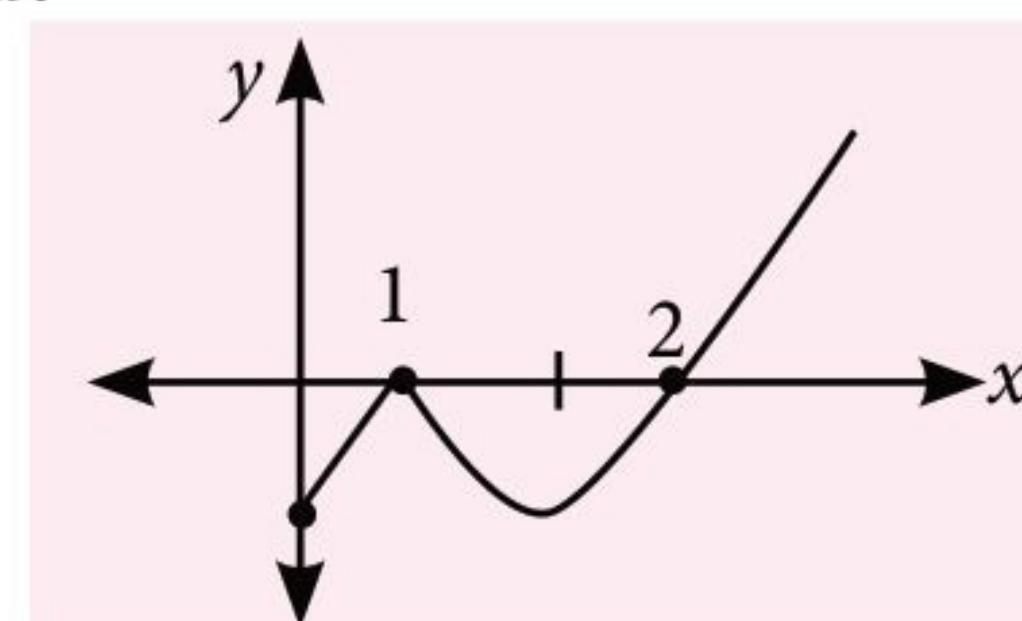
is as shown

When n is even:

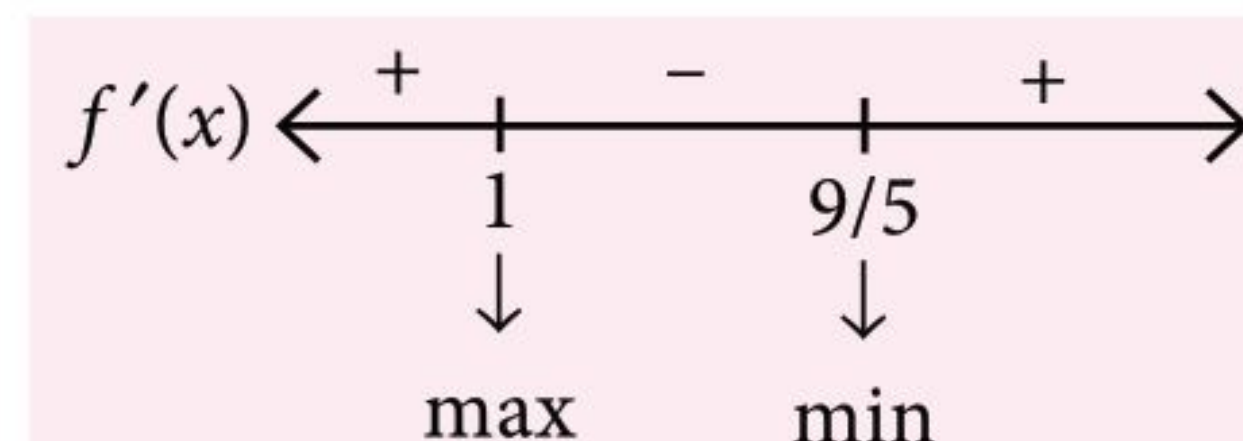
$f(x) = (x - 1)^4(x - 2)^2$
 $f'(x) = 2(x - 1)^3(x - 2)[3x - 5]$



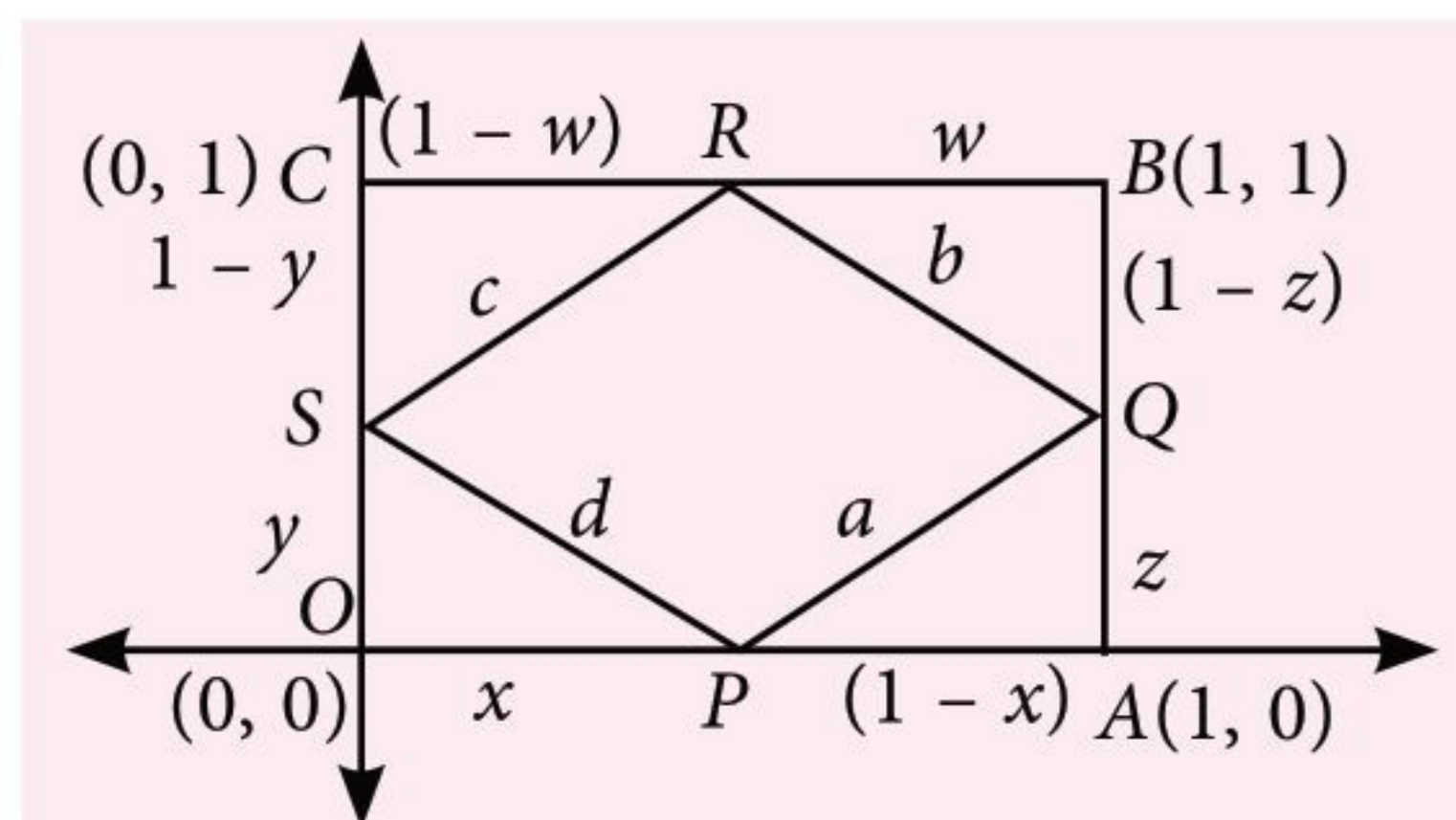
When n is odd:



$f(x) = (x - 1)^4(x - 2)$
 $f'(x) = (x - 1)^3(5x - 9)$



15. (a, b):



$$a^2 + b^2 + c^2 + d^2 = \{x^2 + (1-x)^2\} + \{y^2 + (1-y)^2\} + \{z^2 + (1-z)^2\} + \{w^2 + (1-w)^2\},$$

where $0 \leq x, y, z, w \leq 1$

Consider $f(x) = x^2 + (1-x)^2$, $0 \leq x \leq 1$

$$f'(x) = 2x - 2(1-x)$$

$$f'(x) = 0, \text{ for max/min, when } x = \frac{1}{2}$$

$\therefore f(x)$ is minimum at $x = \frac{1}{2}$, and maximum at $x = 1$.

$$2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$$

$$\begin{aligned} 16. (a, b, d) : \frac{8}{3}a^2 + 2b^2 &= \frac{2}{3}[4a^2 + 3b^2] \\ &= \frac{2}{3}[4(a^2 + b^2) - b^2] \end{aligned}$$

$$|f(1)| \leq 1, |f(0)| \leq 1, f(-1) \leq 1$$

$$\text{Now, } |f(1) - f(0)| \leq 2 \Rightarrow |a + b| \leq 2$$

$$\Rightarrow |f(-1) - f(0)| \leq 2 \Rightarrow |a - b| \leq 2$$

$$\Rightarrow 4(a^2 + b^2) - b^2 = 2((a+b)^2 + (a-b)^2) - b^2 \leq 2 \times 8 - b^2$$

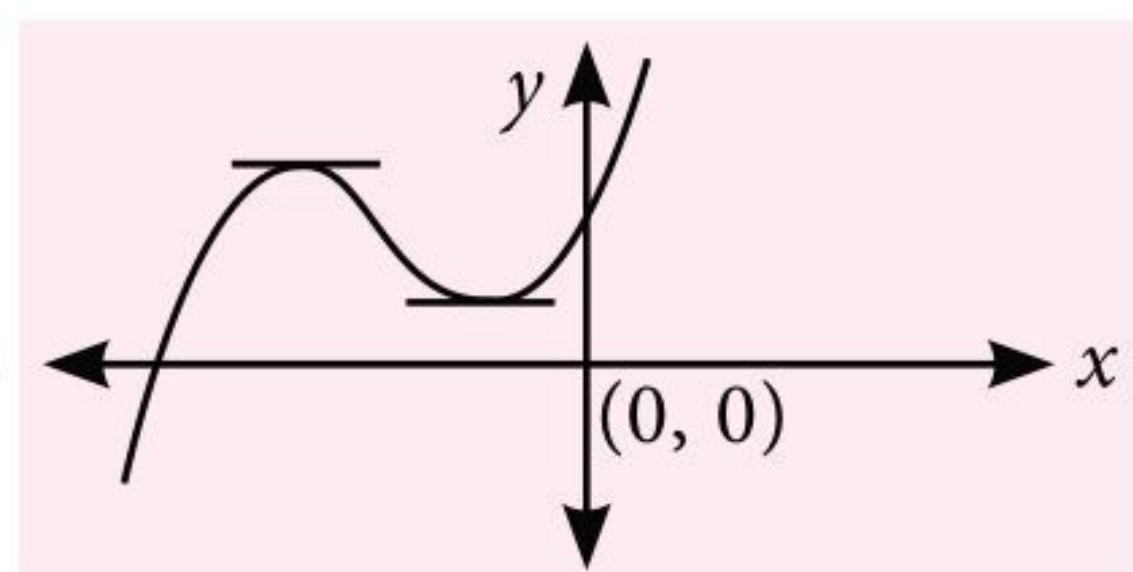
$$\Rightarrow |a + b| = 2, |a - b| = 2$$

$$\text{When } f(1) = 1, f(0) = -1, f(-1) = 1$$

$$\text{or } f(1) = -1, f(0) = 1, f(-1) = -1$$

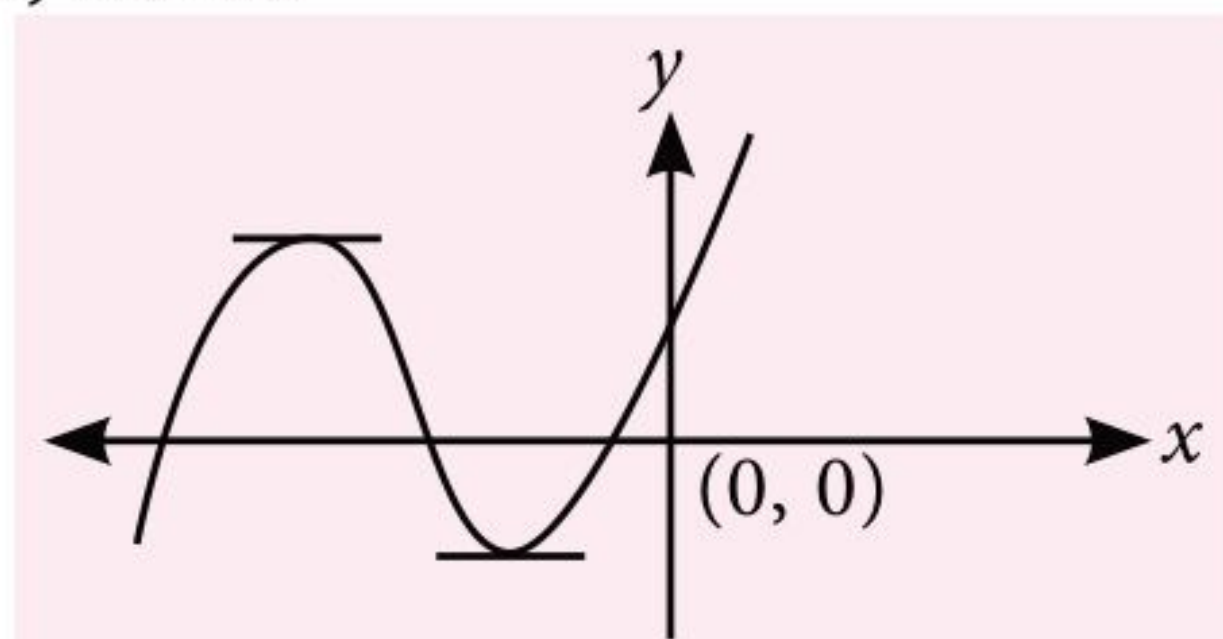
In the second case, we get $b = 0, a = -2, c = 1$

17. (b, c, d) : (a)



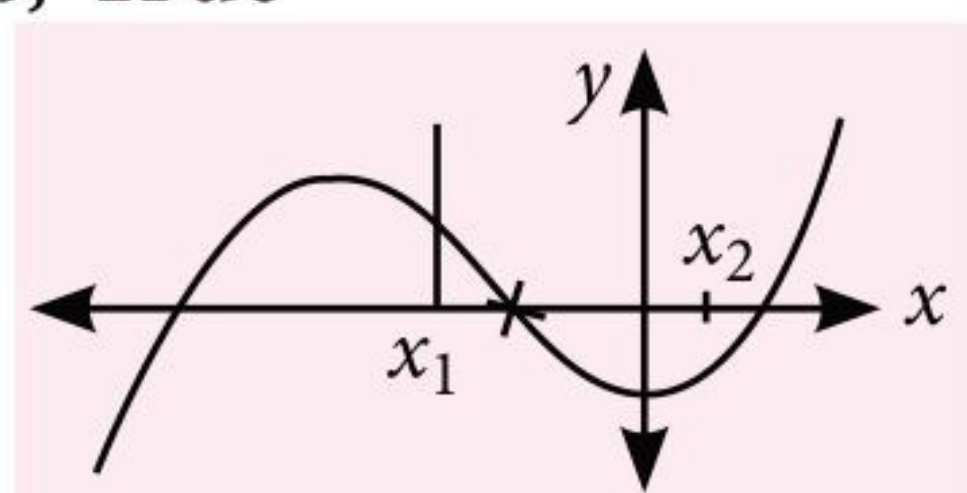
Thus, False

(b)



Thus, True

(c)



Thus, True

$$(d) f''(x) = 2(3x + a) = 0 \Rightarrow x = -\frac{a}{3} \Rightarrow \text{true}$$

$$18. (a, b) : f'(x) \geq 0 \Rightarrow x^4 + x^3 + 3x^2 + kx + 1 \geq 0$$

$$\Rightarrow \left(x^2 + \frac{x}{2}\right)^2 + \left(3 - \frac{1+k^2}{4}\right)x^2 + \left(\frac{kx}{2} + 1\right)^2 \geq 0$$

$$\Rightarrow 3 - \frac{1+k^2}{4} \geq 0 \Rightarrow 3 \geq \frac{1+k^2}{4}$$

$$19. (a, b) : f'_N(x) = \prod_{n=1}^{2018} (x-n)^n$$

Points of minima : $x = 1, 5, 9, \dots, 2017$

Points of maxima : $x = 3, 7, \dots, 2015$

$$20. (a, b, c, d) : y = x^{1/3}(x-1)$$

$$\frac{dy}{dx} = \frac{4}{3}x^{1/3} - \frac{1}{3} \cdot \frac{1}{x^{2/3}} = \frac{1}{3x^{2/3}}[4x-1]$$

Hence, f is increasing for $x > \frac{1}{4}$.

$\therefore x^{2/3}$ is always positive and $x = 1/4$ and the curves has a local minima and f decreasing for $x < \frac{1}{4}$.

$$\text{Now } f'(x) = \frac{4}{3}x^{1/3} - \frac{1}{3} \cdot x^{-2/3}$$

(non existent at $x = 0$, vertical tangent)

$$f''(x) = \frac{4}{9} \cdot \frac{1}{x^{2/3}} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{x^{5/3}}$$

$$= \frac{2}{9x^{2/3}} \left[2 + \frac{1}{x} \right] = \frac{2}{9x^{2/3}} \left[\frac{2x+1}{x} \right]$$

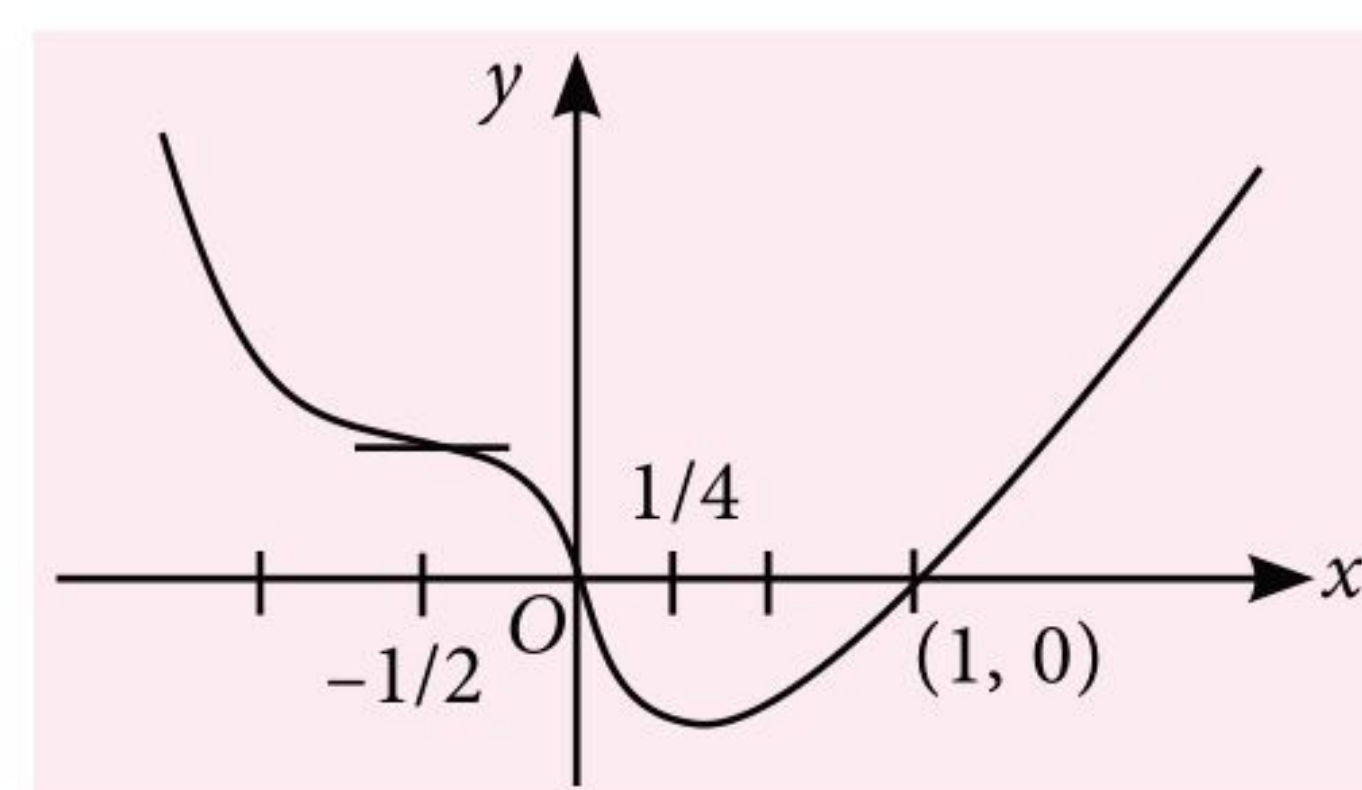
$$\therefore f''(x) = 0 \text{ at } x = -\frac{1}{2} \text{ (inflection point)}$$

Graph of $f(x)$ is as

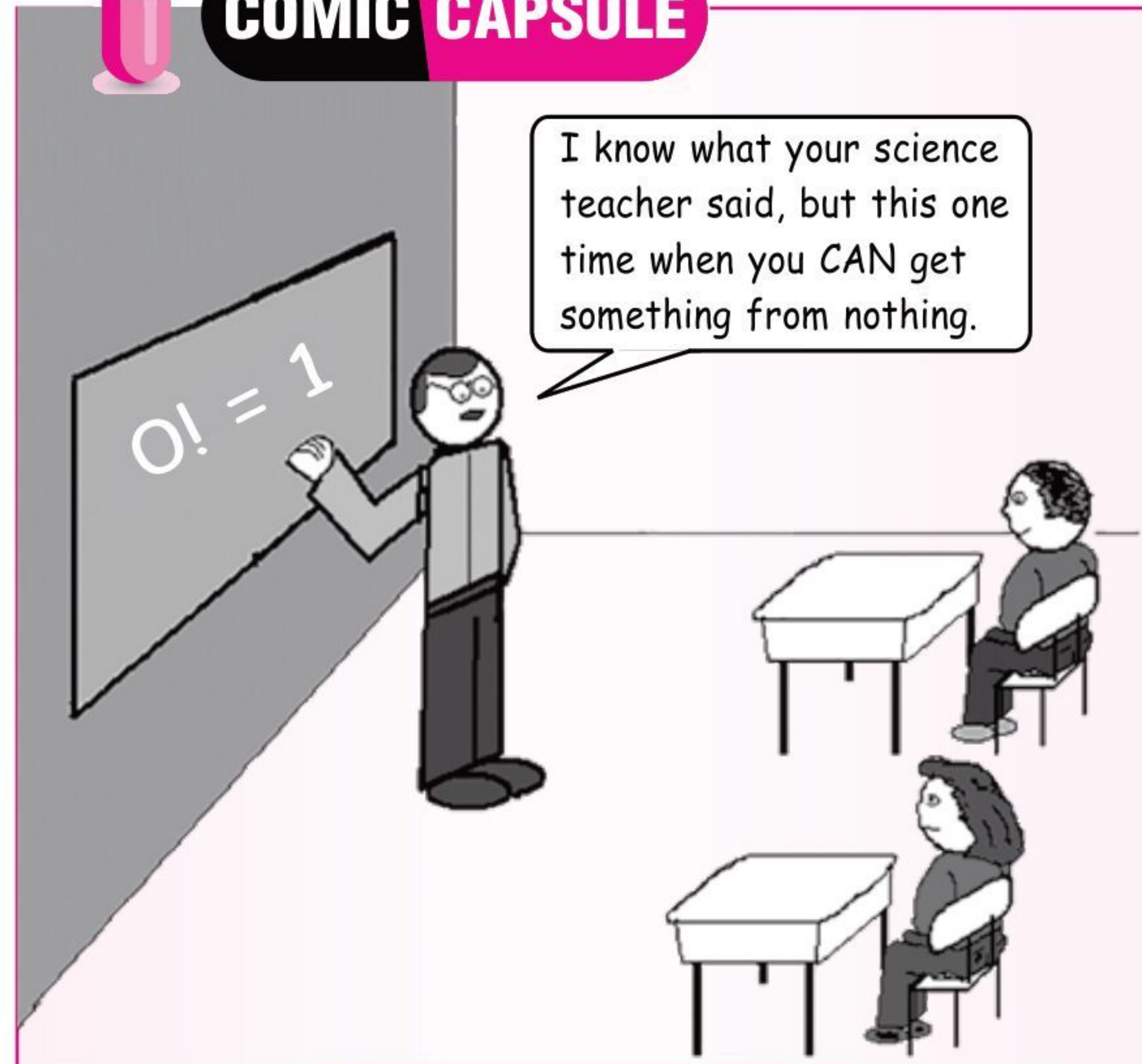
$$A = \left| \int_0^1 (x^{4/3} - x^{1/3}) dx \right|$$

$$= \left| \left[\frac{3}{7}x^{7/3} - \frac{3}{4}x^{4/3} \right]_0^1 \right|$$

$$= \left| \frac{9}{28} \right| = \frac{9}{28}$$



COMIC CAPSULE



CONCEPT MAP

COMPLEX NUMBERS

Class XI

Class XII

THREE DIMENSIONAL GEOMETRY (LINE)

CONCEPT MAP

Complex Number

A number of the form $z = a + ib$, where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$, is called a complex number. Here $\text{Re}(z) = a$ and $\text{Im}(z) = b$.

Algebra of Complex Numbers

Let $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$, then

- $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$
- $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$
- $z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$
- $\frac{z_1}{z_2} = \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i \frac{(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2}$, where $z_2 \neq 0$
- $z_1 = z_2 \Rightarrow a_1 = a_2$ and $b_1 = b_2$

Geometry of Complex Numbers

If z is a variable point and z_1, z_2 are two fixed points in the argand plane, then

- $|z - z_1| = |z - z_2|$ represents perpendicular bisector of the line segment joining z_1 and z_2 .
- $|z - z_1| + |z - z_2| = K$ (a fixed quantity > 0) ... (i)
 - If $K > |z_1 - z_2|$, then (i) represents an ellipse.
 - If $K = |z_1 - z_2|$, then (i) represents the line segment joining z_1 and z_2 .
 - If $K < |z_1 - z_2|$, then (i) does not represent any curve in the argand plane.
- If $K \neq |z_1 - z_2|$, then $|z - z_1| - |z - z_2| = K$ represent a hyperbola with foci at z_1 and z_2 .
- If $K = |z_1 - z_2|$, then $|z - z_1| - |z - z_2| = K$ represents a straight line joining z_1 and z_2 but excluding the line segment joining z_1 and z_2 .

- Triangle ABC with vertices $A(z_1)$, $B(z_2)$ and $C(z_3)$ is

$$\text{equilateral if and only if } \begin{vmatrix} 1 & z_1 & z_2 \\ 1 & z_2 & z_3 \\ 1 & z_3 & z_1 \end{vmatrix} = 0.$$

- The equation of circle whose centre is at point having affix z_0 and radius R is $|z - z_0| = R$.
- The equation of circle whose centre is $-a$ and radius $R = \sqrt{|a|^2 - b}$ is $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$.

Some Basic Terms

If $z = a + ib$, then

- Conjugate: $\bar{z} = a - ib$
- Modulus: $|z| = \sqrt{a^2 + b^2}$
- Argument: $\arg(z) = \tan^{-1}(b/a)$

Properties

- $\overline{(\bar{z})} = z$
- $z = \bar{z} \Leftrightarrow z$ is purely real
- $z + \bar{z} = 0 \Leftrightarrow z$ is purely imaginary
- $z_1 \pm z_2 = \bar{z}_1 \pm \bar{z}_2$
- $z_1 z_2 = \bar{z}_1 \bar{z}_2$
- $(z_1 / z_2) = \bar{z}_1 / \bar{z}_2$; $z_2 \neq 0$
- $(z^n) = (\bar{z})^n$
- $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
- $z\bar{z} = |z|^2 = |\bar{z}|^2$
- $|z^n| = |z|^n$, where $n \in \mathbb{Q}$
- $|z_1 + z_2 + \dots + z_n| \leq |z_1| + |z_2| + \dots + |z_n|$
- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2n\pi \forall n \in \mathbb{I}$
- $\arg(z_1 / z_2) = \arg(z_1) - \arg(z_2) + 2n\pi \forall n \in \mathbb{I}$
- $\arg(z^n) = n \arg(z) + 2n\pi \forall n \in \mathbb{I}$

Different Forms of Complex Numbers

- Polar Form**: $z = a + ib = r(\cos\theta + i\sin\theta) = r \text{cis } \theta$, where $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}(b/a)$
- Euler's Form**: $z = re^{i\theta}$, $\bar{z} = re^{-i\theta}$ where $-\pi < \theta \leq \pi$, θ is the principal argument.

Square Root of a Complex Number

Let $z = a + ib$ be a complex number. So, square root of $z = a + ib$ is defined as,

$$\sqrt{a + ib} = \pm \left\{ \sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} + a)} + i \sqrt{\frac{1}{2}(\sqrt{a^2 + b^2} - a)} \right\}$$

To find the square root of $a - ib$, replace i by $-i$ in the above result.

Cube Roots of Unity

Let $x = (1)^{1/3} \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$
or $x = 1, \omega, \omega^2$

Note: (i) $\omega^3 = 1$ (ii) $1 + \omega + \omega^2 = 0$

(iii) $1 + \omega^n + \omega^{2n} = \begin{cases} 0, & \text{if } n \text{ is not a multiple of } 3 \\ 3, & \text{if } n \text{ is a multiple of } 3 \end{cases}$

Direction Cosines and Direction Ratios

- Direction cosines**: If a line L makes an angle α, β, γ ($0 \leq \alpha, \beta, \gamma \leq \pi$) with the positive direction of x, y, z axes respectively then $(l, m, n) = (\cos\alpha, \cos\beta, \cos\gamma)$ are the direction cosines of the line L and $l^2 + m^2 + n^2 = 1$.

$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \quad \cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}} \quad \cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

D.C.s of axes:
x-axis: $(1, 0, 0)$
y-axis: $(0, 1, 0)$
z-axis: $(0, 0, 1)$

- Direction ratios**: Any three numbers proportional to (l, m, n) are called direction ratios. We denote them by (a, b, c) .

$$\therefore \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k$$

Shortest Distance between Two Lines

- Vector form**: Let l_1 and l_2 be two skew lines whose equations are $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ respectively. Let $\overline{PQ}$ be the shortest distance vector between l_1 and l_2 . Then, $PQ = \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}$

If lines are parallel, then

$$\vec{b}_1 = \vec{b}_2 = \vec{b} \text{ and } d = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

- Cartesian form**: Let two skew lines be

$$l_1: \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } l_2: \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

The shortest distance between l_1 and l_2 is given by

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

Equation of Line in Different Forms

- Equation of a line in space**:
Vector form: $\vec{r} = \vec{a} + \lambda \vec{b}$, $\lambda \in \mathbb{R}$, where $A(\vec{a})$ is passing point and $\vec{b}$ is the parallel vector.

$$\text{Cartesian form: } \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}, \text{ where } (x_1, y_1, z_1) \text{ is passing point and } (a_1, b_1, c_1) \text{ are direction ratios of the line.}$$

- Line passing through two points**:

Vector form: $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, $\lambda \in \mathbb{R}$, where $A(\vec{a})$ and $B(\vec{b})$ are the passing points.

$$\text{Cartesian form: } \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}, \text{ where } A(x_1, y_1, z_1) \text{ and } B(x_2, y_2, z_2) \text{ are the passing points.}$$

Angle Between Two Lines

- Vector form**: Let $\vec{r}_1 = \vec{a}_1 + \lambda \vec{b}_1$ & $\vec{r}_2 = \vec{a}_2 + \mu \vec{b}_2$ be the two lines then $\cos\theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| |\vec{b}_2|}$.

- Cartesian form**: Let $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ be the two lines then

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Coplanarity of Two Lines

- Vector form**: Two lines $\vec{r}_1 = \vec{a}_1 + \lambda_1 \vec{n}_1$, $\vec{r}_2 = \vec{a}_2 + \lambda_2 \vec{n}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{n}_1 \times \vec{n}_2) = 0$.

- Cartesian form**: Two lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$$

beat the TIME TRAP



Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is

- (a) $8\sqrt{10}$ (b) $6\sqrt{10}$ (c) $12\sqrt{10}$ (d) $12\sqrt{15}$

2. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$$

If $B - 2A = 100\lambda$, then λ is equal to

- (a) 496 (b) 232 (c) 248 (d) 464

3. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOPQ is equilateral, O being the centre. Then the eccentricity e satisfies

- (a) $1 < e < \frac{2}{\sqrt{3}}$ (b) $e = \frac{2}{\sqrt{2}}$
(c) $e = \frac{\sqrt{3}}{2}$ (d) $e > \frac{2}{\sqrt{3}}$

4. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is

- (a) $\frac{2}{3}$ (b) $\frac{4 - \sqrt{7}}{3}$
(c) $\frac{4 - \sqrt{5}}{3}$ (d) $\frac{\sqrt{7} - 2}{3}$

5. The minimum value of the function $f(x) = 2|x - 1| + |x - 2|$ is

- (a) 0 (b) 1 (c) 2 (d) 3

6. If $I = \lim_{x \rightarrow 0} \sin \left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2} \right)$, then limit

- (a) does not exist (b) exists and equals 1
(c) exists and equals 0 (d) exists and equals $\frac{1}{2}$

7. If a straight line passes through the point (α, β) and the portion of the line intercepted between the axes is divided equally at that point, then $\frac{x}{\alpha} + \frac{y}{\beta}$ is

- (a) 0 (b) 1 (c) 2 (d) 4

8. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ is

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

9. An insurance company insured 1800 car drivers, 1500 bike drivers and 5000 truck drivers. The probability of a car, a bike and a truck driver meeting with an accident is 0.02, 0.03 and 0.06 respectively. If one of the insured person meets with an accident, then find the probability that he is a bike driver.

- (a) $\frac{15}{127}$ (b) $\frac{15}{129}$ (c) $\frac{14}{127}$ (d) $\frac{13}{129}$

10. If the system of linear equations $x - 4y + 7z = g$, $3y - 5z = h$, $-2x + 5y - 9z = k$ is consistent, then

- (a) $g + h + k = 0$ (b) $2g + h + k = 0$
(c) $g + h + 2k = 0$ (d) $g + 2h + k = 0$

SECTION-II

Numerical Answer Type

11. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B , then the value of $m \cdot n$ is _____.

12. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

13. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at $x =$ _____.

14. The integral $\int_0^2 ||x-1| - x| dx$ is equal to _____.

15. Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$, $\lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k})$, $\mu \in R$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P , then $3(\alpha + \beta + \gamma)$ equals _____.

SOLUTIONS

1. (c): Let AC be the vertical pole and B is point of division on it.

In $\triangle ABD$,

$$\tan \theta = \frac{3x}{18} = \frac{x}{6} \quad \dots(i)$$

$$\text{In } \triangle ACD, \tan 2\theta = \frac{10x}{18} = \frac{5x}{9}$$

$$\Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{5x}{9} \Rightarrow \frac{2 \times \frac{x}{6}}{1 - \frac{x^2}{36}} = \frac{5x}{9} \quad (\text{Using (i)})$$

$$\Rightarrow \frac{x}{3} \times \frac{36}{(36 - x^2)} = \frac{5x}{9} \Rightarrow 108x = 180x - 5x^3$$

$$\Rightarrow 5x^3 - 72x = 0 \Rightarrow x(5x^2 - 72) = 0$$

$$\Rightarrow x^2 = \frac{72}{5} \quad (\because x \neq 0)$$

$$\Rightarrow x = 6\sqrt{\frac{2}{5}} \quad (\because x \text{ can't be negative})$$

$$\text{So, height of pole} = 10x = 10 \times 6\sqrt{\frac{2}{5}} = 12\sqrt{10} \text{ m}$$

2. (c): Let $S = 1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$

The sum of first 20 terms is

$$A = (1^2 + 2^2 + \dots + 20^2) + (2^2 + 4^2 + \dots + 20^2) \\ = \frac{20 \cdot 21 \cdot 41}{6} + 4 \cdot \frac{10 \cdot 11 \cdot 21}{6} = \frac{20 \cdot 21}{6} (41 + 22) = 4410$$

$$B = 1^2 + 2 \cdot 2^2 + \dots + 2 \cdot 40^2 \\ = (1^2 + 2^2 + \dots + 40^2) + (2^2 + 4^2 + \dots + 40^2)$$

$$= \frac{40 \cdot 41 \cdot 81}{6} + \frac{4 \cdot 20 \cdot 21 \cdot 41}{6} = 33620$$

$$B - 2A = 33620 - 8820 = 24800$$

$$\therefore 100\lambda = 24800 \Rightarrow \lambda = 248$$

3. (d): Let $OM = a \sec \theta$,
 $PM = b \tan \theta$

$$\tan 30^\circ = \frac{PM}{OM} = \frac{b \sin \theta}{a}$$

$$\Rightarrow \sin \theta = \frac{a}{b\sqrt{3}} \Rightarrow \sin^2 \theta = \frac{a^2}{3b^2}$$

$$\because 0 \leq \sin^2 \theta \leq 1 \quad \therefore 0 \leq \frac{a^2}{3b^2} \leq 1 \quad \left[\because \frac{b^2}{a^2} = e^2 - 1 \right]$$

$$\Rightarrow \frac{3b^2}{a^2} \geq 1 \Rightarrow 3(e^2 - 1) \geq 1 \Rightarrow e^2 \geq \frac{4}{3} \Rightarrow e \geq \frac{2}{\sqrt{3}}$$

4. (b): Since $f(x)$ is a polynomial function.

$\therefore$ It is continuous and differentiable in $[0, 1]$

Here, $f(0) = 11$, $f(1) = 1 - 4 + 8 + 11 = 16$

$$f'(x) = 3x^2 - 8x + 8$$

$$\therefore f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{16 - 11}{1} = 3c^2 - 8c + 8$$

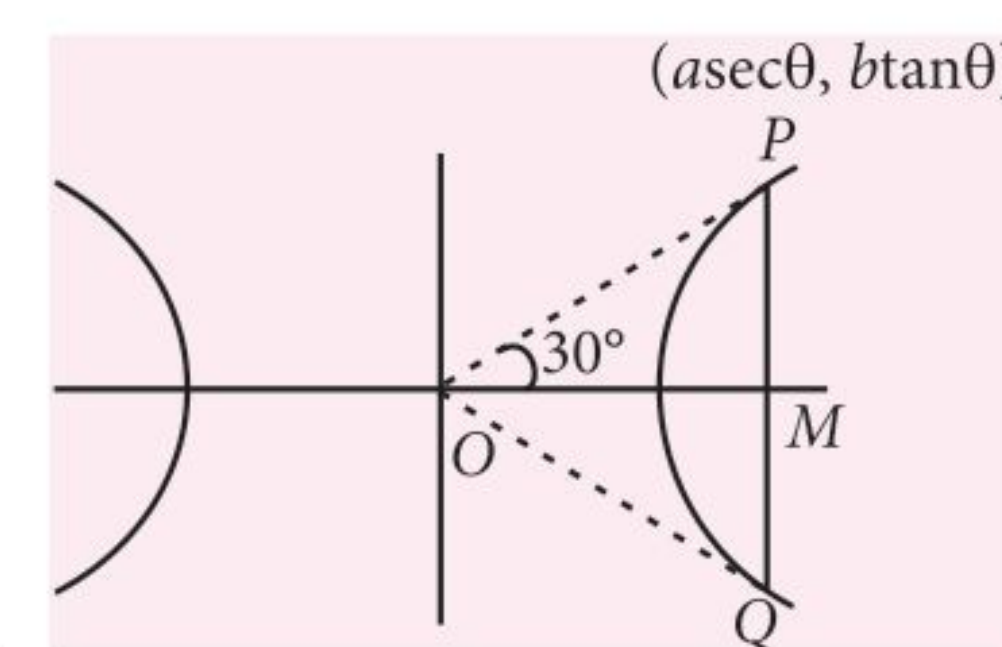
$$\Rightarrow 3c^2 - 8c + 3 = 0 \Rightarrow c = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

$$\therefore c = \frac{4 - \sqrt{7}}{3} \in (0, 1)$$

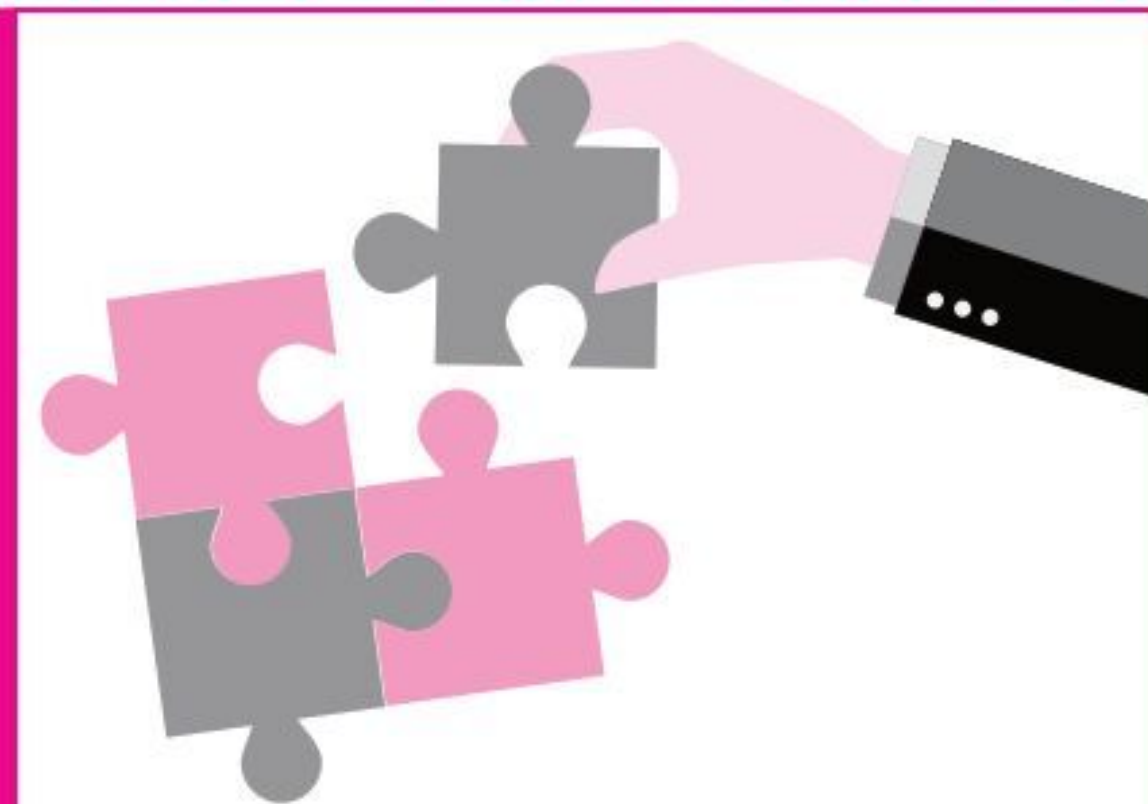
5. (b): $f(x) = 2|x - 1| + |x - 2|$

$$f(x) = \begin{cases} -2(x-1) - (x-2), & x < 1 \\ 2(x-1) - (x-2), & 1 \leq x < 2 \\ 2(x-1) + (x-2), & x \geq 2 \end{cases} = \begin{cases} 4 - 3x, & x < 1 \\ x, & 1 \leq x < 2 \\ 3x - 4, & x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} -3, & x < 1 \\ 1, & 1 \leq x < 2 \\ 3, & x \geq 2 \end{cases}$$



PUZZLE CORNER
ANSWER · NOVEMBER 2022



1- 2	3	108x 6	5+ 1	4	50x 5
15+ 4	1- 1	3	6	5	2
1	2	1- 4	5	3- 3	6
6	4	2- 5	3	3+ 2	1
2- 3	4- 5	1	2+ 2	6	4
5	4- 6	2	4	3+ 1	3

If $f'(x)$ changes sign from negative to positive at any point, then $f(x)$ is minimum at that point.

$\therefore$ Minimum value of $f(x)$ is

$$f(1) = 0 + |-1| = 1.$$

$$\begin{aligned} 6. \text{ (c) : } I &= \lim_{x \rightarrow 0} \sin \left(\frac{e^x - x - 1 - \frac{x^2}{2}}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \sin \left[\frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right) - x - 1 - \frac{x^2}{2}}{x^2} \right] \\ &= \lim_{x \rightarrow 0} \sin \left(\frac{\frac{x^3}{3!} + \frac{x^4}{4!} + \dots}{x^2} \right) = \lim_{x \rightarrow 0} \sin \left[\frac{x}{3!} + \frac{x^2}{4!} + \dots \right] = 0 \end{aligned}$$

Thus, limit exists and equals to 0.

7. (c) : Let the line be AB i.e., $\frac{x}{a} + \frac{y}{b} = 1$... (1)

$\therefore A(a, 0), B(0, b)$

$\therefore (\alpha, \beta)$ is mid point of AB

$$\therefore \alpha = \frac{a}{2}, \beta = \frac{b}{2}$$

$$\therefore a = 2\alpha, b = 2\beta$$

$\therefore$ From (1),

$$\frac{x}{2\alpha} + \frac{y}{2\beta} = 1 \Rightarrow \frac{x}{\alpha} + \frac{y}{\beta} = 2$$

8. (d) : A vector normal to first plane is

$$\vec{n}_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$$

A vector normal to second plane is

$$\vec{n}_2 = (\hat{i} - \hat{j}) \times (\hat{i} + \hat{k}) = -\hat{j} + \hat{k} - \hat{i}$$

Since, $\vec{a}$ will be parallel to $\vec{n}_1 \times \vec{n}_2 = \hat{i} - \hat{j}$

Let θ be the angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\therefore \cos \theta = \frac{(\hat{i} - \hat{j}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 1^2} \sqrt{1^2 + 2^2 + 2^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

9. (a) : Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 : insured person is a car driver

E_2 : insured person is a bike driver

E_3 : insured person is a truck driver

A : person meets with an accident

Total number of insured persons

$$= 1800 + 1500 + 5000 = 8300$$

$$\therefore P(E_1) = \frac{1800}{8300} = \frac{18}{83}, P(E_2) = \frac{1500}{8300} = \frac{15}{83},$$

$$P(E_3) = \frac{5000}{8300} = \frac{50}{83}$$

$$P(A|E_1) = P(\text{a car driver meeting with an accident}) = 0.02$$

$$P(A|E_2) = 0.03, P(A|E_3) = 0.06$$

We want to find $P(E_2|A)$

By Bayes' theorem, we have

$$\begin{aligned} P(E_2|A) &= \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)} \\ &= \frac{\frac{15}{83} \times 0.03}{\frac{18}{83} \times 0.02 + \frac{15}{83} \times 0.03 + \frac{50}{83} \times 0.06} = \frac{0.45}{3.81} = \frac{15}{127} \end{aligned}$$

$$10. \text{ (b) : We have, } \Delta = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix}$$

$$= 1(-27 + 25) + 0 - 2(20 - 21) = -2 + 2 = 0$$

$\therefore$ System of equations will be consistent if

$$\Delta_x = \Delta_y = \Delta_z = 0$$

$$\therefore \Delta_z = \begin{vmatrix} 1 & -4 & g \\ 0 & 3 & h \\ -2 & 5 & k \end{vmatrix} = 0$$

$$\Rightarrow 1(3k - 5h) - 2(-4h - 3g) = 0$$

$$\Rightarrow 3k - 5h + 8h + 6g = 0 \Rightarrow 2g + h + k = 0$$

$$\text{Similarly, } \Delta_x = \Delta_y = 0 \Rightarrow 2g + h + k = 0$$

11. (28) : We know that number of subsets of a set containing n elements is 2^n .

According to question,

$$2^m = 2^n + 112$$

$$\Rightarrow 2^m - 2^n = 112$$

$$\Rightarrow 2^n(2^{m-n} - 1) = 16 \times 7 = 2^4 \times (8 - 1)$$

$$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

$$\text{Thus, } m - n = 3 \text{ and } n = 4 \Rightarrow m = 7$$

$$\text{So, } m \times n = 7 \times 4 = 28$$

12. (120) : There are 2 vowels E, E and 4 consonants L, T, T, R in the word "LETTER".

Total number of possible words that can be formed

$$\text{from all the letters of the word "LETTER"} = \frac{6!}{2!2!} = 180$$

Now, if we consider the vowels EE as single letter, then

$$\text{number of possible words} = \frac{5!}{2!} = 60$$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (a) 2. (c) 3. (b) 4. (c) 5. (c)
6. (d) 7. (b,c) 8. (a,b,c,d) 9. (a,d) 10. (c,d)
11. (a,d) 12. (a,b,c) 13. (b) 14. (b) 15. (b)
16. (c) 17. (5) 18. (7) 19. (3) 20. (3)

Now, required number of words = Total number of words – number of words in which vowels come together = $180 - 60 = 120$

13. (3) : Let $f(x) = ax^3 + bx^2 + cx + d$

$$f'(x) = 3ax^2 + 2bx + c$$

$$f''(x) = 6ax + 2b$$

$$\text{Now, } f'(-1) = 3a - 2b + c = 0 \quad \dots(i)$$

$$f''(1) = 6a + 2b = 0 \quad \dots(ii)$$

$$\text{Also, } f(-1) = 10 \Rightarrow -a + b - c + d = 10 \quad \dots(iii)$$

$$\text{and } f(1) = -6 \Rightarrow a + b + c + d = -6 \quad \dots(iv)$$

Solving (i), (ii), (iii) and (iv), we get

$$a = 1, b = -3, c = -9 \text{ and } d = 5$$

$$\therefore f(x) = x^3 - 3x^2 - 9x + 5$$

$$\Rightarrow f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3)$$

$$= 3(x - 3)(x + 1)$$

$$\text{Now, } f'(x) = 0 \Rightarrow x = 3, -1$$

$\therefore$ Local minima exist at $x = 3$.

14. (1.5) : Let $I = \int_0^2 ||x-1| - x| dx$

$$= \int_0^1 |1-x-x| dx + \int_1^2 |x-1-x| dx$$

$$= \int_0^1 |1-2x| dx + \int_1^2 |-1| dx$$

Sign of $f'(x)$		
+	-	+
-1		3

$$= \int_0^{1/2} (1-2x) dx + \int_{1/2}^1 (2x-1) dx + \int_1^2 dx$$

$$= (x - x^2)_0^{1/2} + (x^2 - x)_{1/2}^1 + (x)_1^2$$

$$= \frac{1}{2} - \frac{1}{4} + (1-1) - \left(\frac{1}{4} - \frac{1}{2} \right) + 2 - 1$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{4} + \frac{1}{2} + 1 = 2 - \frac{1}{2} = \frac{3}{2} = 1.5$$

$$\text{15. (5) : Normal to the plane} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow \text{d.r.'s of normal} = \langle -1, 1, 1 \rangle$$

$$\text{So, equation of plane is, } -1(x-1) + 1(y-0) + 1(z-0) = 0$$

$$\Rightarrow x - y - z - 1 = 0$$

If (α, β, γ) be the foot of perpendicular of $M(1, 0, 1)$ on the plane, then we have

$$\frac{\alpha-1}{1} = \frac{\beta-0}{-1} = \frac{\gamma-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$$\Rightarrow \alpha = \frac{4}{3}, \beta = \frac{-1}{3}, \gamma = \frac{2}{3}$$

$$\therefore 3(\alpha + \beta + \gamma) = 4 - 1 + 2 = 5$$

UNSCRAMBLE ME

Unscramble the words given in column I and match them with their explanations in column II.

Column I

1. RCTEOV
2. YTOTALUGO
3. ONIT
4. EGANL
5. DTINOINUC
6. OMOTUCE
7. EINYTQAUIL
8. ANREG
9. FANTSIRUQEI
10. DNOMIA

Column II

- (a) A function $f : A \rightarrow B$ is such that there exists atleast single element in B having no pre-image in A.
- (b) Phrases like 'There exists' and 'For all' which are frequently found in mathematical statements.
- (c) A physical quantity which have both magnitude as well as direction.
- (d) It is the possible result of a random experiment.
- (e) A compound statement which is always true for all truth values of its component statements.
- (f) A mathematical statement involving the symbol $>$, $<$, $\leq$ or $\geq$.
- (g) The set of all first elements of the ordered pairs in relation R from set X to set Y.
- (h) The measure of rotation of a ray about its initial point.
- (i) The process of drawing a valid general result from a particular result.
- (j) The set of all second elements of the ordered pairs in relation R from set X to set Y.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month.
Winners' names will be published in next issue.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then find the value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r$.

(Ridhima Paul, Maharashtra)

Ans. It is given that α, β are roots of the given quadratic equation $375x^2 - 25x - 2 = 0$

$$\therefore \alpha + \beta = \frac{25}{375} \text{ and } \alpha\beta = \frac{-2}{375}$$

$$\text{Now, } \lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r$$

$$= (\alpha + \alpha^2 + \dots \infty) + (\beta + \beta^2 + \dots \infty)$$

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{\alpha + \beta - 2\alpha\beta}{1 - (\alpha + \beta) + \alpha\beta}$$

$$= \frac{\frac{25}{375} - 2\left(-\frac{2}{375}\right)}{1 - \left(\frac{25}{375}\right) + \left(-\frac{2}{375}\right)} = \frac{1}{12}$$

$$2. \text{ If } f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous at $x = 0$, then find the ordered pair (p, q) .

(Ankit Jain, Delhi)

Ans. Since, it is given that $f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0) \quad \dots(i)$$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$= \lim_{x \rightarrow 0^-} (p+1) \frac{\sin(p+1)x}{(p+1)x} + 1$$

$$= (p+1) + 1 = p+2 \quad \dots(ii)$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sqrt{x+1} - 1}{x} \left(\frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{2\sqrt{x+1}} \quad (\text{Applying L' Hospital's rule})$$

$$= \frac{1}{2} \quad \dots(iii)$$

$$\text{Also, } f(0) = q \quad \dots(iv)$$

From (i), (ii), (iii) and (iv), we get

$$p+2 = \frac{1}{2} = q \quad \therefore p = -\frac{3}{2}, q = \frac{1}{2}.$$

3. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e , then show that $4e^4 - 24e^2 + 35 = 0$.

(Mayank, Haryana)

Ans. Let the equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since it passes through the point $(4, -2\sqrt{3})$. [Given]

$$\frac{16}{a^2} - \frac{12}{a^2(e^2-1)} = 1 \quad \left[\because e^2 = \left(1 + \frac{b^2}{a^2}\right) \right] \quad \dots(i)$$

It is given that the equation of directrix is $x = \frac{4}{\sqrt{5}}$.

$$\therefore \frac{a}{e} = \frac{4}{\sqrt{5}} \Rightarrow a^2 = \frac{16}{5}e^2 \quad \dots(ii)$$

$$\text{From (i) and (ii), we get } \frac{16}{\frac{16}{5}e^2} - \frac{12}{\frac{16}{5}e^2(e^2-1)} = 1$$

$$\Rightarrow 16 \left(\frac{5}{16e^2} \right) - \frac{12}{(e^2-1)} \left(\frac{5}{16e^2} \right) = 1$$

$$\Rightarrow \frac{5}{16e^2} \left(\frac{16e^2 - 12}{e^2 - 1} \right) = 1$$

$$\Rightarrow 80e^2 - 140 = 16e^4 - 16e^2$$

$$\Rightarrow 16e^4 - 96e^2 + 140 = 0 \Rightarrow 4e^4 - 24e^2 + 35 = 0$$



MONTHLY TEST DRIVE CLASS XI

ANSWER

KEY

- | | | | | |
|---------|-------------|-------------|------------|-------------|
| 1. (c) | 2. (c) | 3. (a) | 4. (b) | 5. (c) |
| 6. (b) | 7. (b,c,d) | 8. (b,d) | 9. (a,b,c) | 10. (a,b,c) |
| 11. (b) | 12. (a,b,c) | 13. (b,c,d) | 14. (d) | 15. (b) |
| 16. (d) | 17. (6) | 18. (5) | 19. (6) | 20. (9) |



CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-5

Conic Sections

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

- The radius of the circle represented by the equation $3x^2 + 3y^2 + \lambda xy + 9x + (\lambda - 6)y + 3 = 0$ is
(a) $3/2$ units (b) $\frac{\sqrt{17}}{2}$ units
(c) $2/3$ unit (d) None of these
- The centre of the circle $(x - a)(x - c) + (y - b)(y - d) = 0$ is
(a) (a, a) (b) (c, d)
(c) $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$ (d) None of these
- If $(x, 3)$ and $(3, 5)$ are the extremities of a diameter of a circle with centre at $(2, y)$, then the values of x and y are
(a) $x = 1, y = 4$ (b) $x = 4, y = 1$
(c) $x = 8, y = 2$ (d) None of these
- The value of p for which the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ represents a circle of radius 5 units is
(a) 3 (b) -3
(c) both (a) and (b) (d) Neither (a) nor (b)
- If the focus of a parabola is $(0, -3)$ and its directrix is $y = 3$, then its equation is
(a) $x^2 = -12y$ (b) $x^2 = 12y$
(c) $y^2 = -12x$ (d) $y^2 = 12x$
- If the parabola $y^2 = 4ax$ passes through the point $(3, 2)$, then the length of its latus rectum is
(a) $2/3$ (b) $4/3$ (c) $1/3$ (d) 4
- Find the coordinates of the focus of the parabola $y^2 = 12x$.
(a) $(0, 3)$ (b) $(-3, 0)$ (c) $(0, -3)$ (d) $(3, 0)$
- The equation of the directrix of the parabola $(x - 1)^2 = 2(y - 2)$ is
(a) $2y + 3 = 0$ (b) $2x + 1 = 0$
(c) $2x - 1 = 0$ (d) $2y - 3 = 0$
- If e is the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a < b$), then

- (a) $b^2 = a^2(1 - e^2)$ (b) $a^2 = b^2(1 - e^2)$
 (c) $a^2 = b^2(e^2 - 1)$ (d) $b^2 = a^2(e^2 - 1)$
10. The eccentricity of the ellipse $12x^2 + 7y^2 = 84$ is equal to
 (a) $\frac{\sqrt{5}}{7}$ (b) $\sqrt{\frac{5}{12}}$ (c) $\frac{\sqrt{5}}{12}$ (d) $\frac{5}{7}$
11. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is
 (a) 4 (b) 3 (c) 8 (d) $\frac{4}{\sqrt{3}}$
12. Equation of the ellipse whose foci are (4, 0) and (-4, 0) and $e = 1/3$ is
 (a) $\frac{x^2}{9} + \frac{y^2}{8} = 16$ (b) $\frac{x^2}{8} + \frac{y^2}{9} = 16$
 (c) $\frac{x^2}{9} + \frac{y^2}{8} = 32$ (d) None of these
13. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is
 (a) $x^2 - y^2 = 32$ (b) $\frac{x^2}{4} - \frac{y^2}{9} = 1$
 (c) $2x^2 - 3y^2 = 7$ (d) None of these
14. The eccentricity of the hyperbola $x^2 - 4y^2 = 1$ is
 (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{5}}$
15. Find the centre and radius of circle $x^2 + y^2 - 2x + 4y = 8$.
 (a) (1, 2), $\sqrt{13}$ (b) (1, -2), $\sqrt{13}$
 (c) (-1, -2), $\sqrt{13}$ (d) (2, -1), 13
16. Find the equation of a parabola with vertex at origin and axis along x -axis and passing through the point (2, 8).
 (a) $y^2 = 32x$ (b) $y^2 = -32x$
 (c) $y^2 = 16x$ (d) $y^2 = -16x$
17. If e is the eccentricity of a hyperbola $3x^2 - 3y^2 = 25$, then e^2 is
 (a) 4 (b) 5 (c) 3 (d) 2
18. The equation of the ellipse whose centre is at the origin and the x -axis is the major axis and which passes through the points (-3, 1) and (2, -2) is
 (a) $5x^2 + 3y^2 = 32$ (b) $3x^2 + 5y^2 = 32$
 (c) $5x^2 - 3y^2 = 32$ (d) $3x^2 + 5y^2 + 32 = 0$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. **Assertion (A)** : The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10, respectively.

Reason (R) : The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.

20. As all points of parabola are equidistant from a fixed line and a fixed point.

Assertion (A) : The fixed line is called the directrix of the parabola.

Reason (R) : The fixed point is called the focus.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

OR

Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

22. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.

23. Find the equation of the ellipse whose vertices are $(\pm 13, 0)$ and foci are $(\pm 5, 0)$.

OR

If the eccentricity of an ellipse is $5/8$ and the distance between its foci is 10, then find the length of latus rectum of the ellipse.

24. Find the eccentricity of the ellipse : $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

25. What is the eccentricity of the curve $4x^2 + y^2 = 100$?

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. A circle of radius 2 units lies in the first quadrant and touches both the coordinate axes. Find the equation of the circle with centre at (6, 5) and touching the above circle externally.

OR

Find the equation of the image of the circle $x^2 + y^2 + 8x - 16y + 64 = 0$ in the line mirror $x = 0$.

27. Find the equation of the circle whose centre is at the point (4, 5) and which passes through the centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.

OR

Find the equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directrices.

28. Find the equation of the ellipse whose foci are (2, 3), $(-2, 3)$ and whose semi-minor axis is $\sqrt{5}$.
29. Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, length of latus rectum is 5 and the centre is (0, 0) and axes as coordinate axes.
30. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola if its eccentricity is 2.
31. Show that the set of all points such that the difference of their distances from (4, 0) and $(-4, 0)$ is always equal to 2 represent a hyperbola.

OR

If P is any point on a hyperbola and N is the foot of the perpendicular from P on the transverse axis,

then prove that $\frac{(PN)^2}{(AN)(A'N)} = \frac{b^2}{a^2}$,

Where A & A' are the vertices of the hyperbola.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Find the vertex, focus, directrix, axis and latus-rectum of the parabola $y^2 = 4x + 4y$.
33. Find the eccentricity, foci and the length of the latus rectum of the ellipse $x^2 + 4y^2 + 8y - 2x + 1 = 0$.

OR

Find the length of the major axis, the minor axis and latus rectum, the coordinates of the vertices and the foci, the eccentricity and the equations of the directrices of the conic section $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

34. Find the vertex, focus and directrix of the parabola $4y^2 + 12x - 12y + 39 = 0$.

OR

Find the equation of the parabola with vertex $(2, -3)$ and focus $(0, 5)$.

35. Show that the equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola. Find the coordinates of the centre, length of the axes of the hyperbola.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case study 1:** Ram went to a nursery to buy new plants for his park. There he saw southern crabapple plant. He broke one leaf of southern crabapple plant and he found that the shape of leaf was elliptical.

The length of major axis and minor axis are 10 cm and 8 cm respectively. Consider the centre of ellipse is at origin.

Based on the above information, answer the following questions.

- Find the equation of curve.
- Find the distance between foci.
- Find the ends of latus rectum.

OR

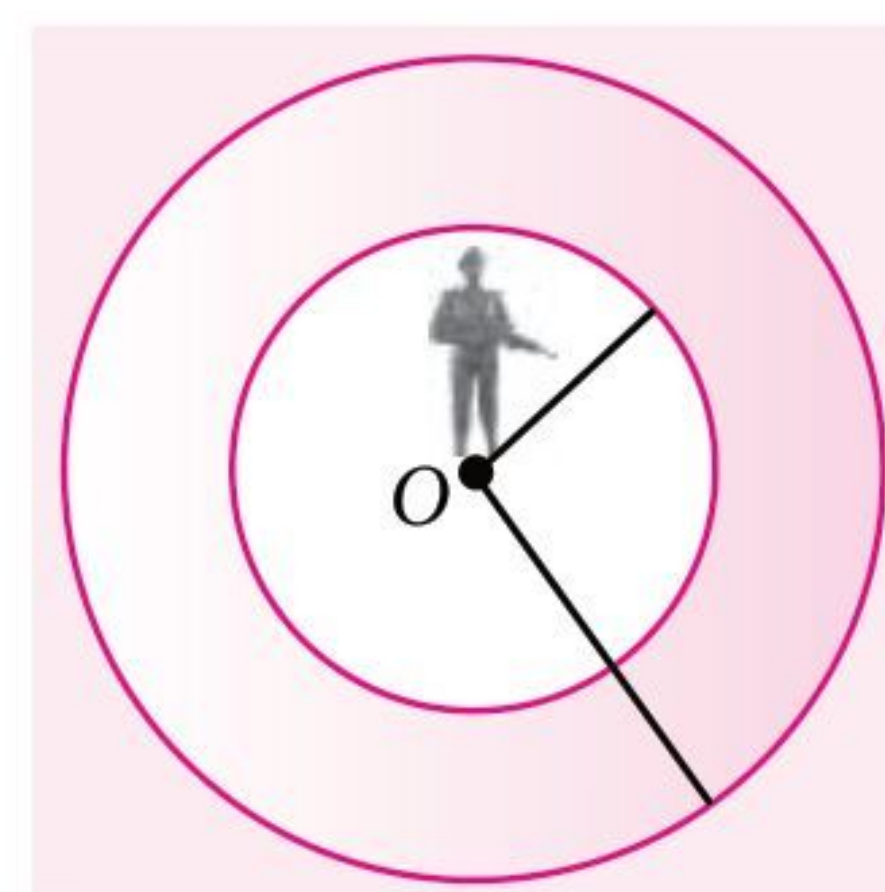
Find the equation of directrix.

37. **Case study 2:** The townsfolk of a city erected a statue of a great soldier in the center of the city. To accomplish so, they build a circular park in the center of the city. The park was mostly covered by grass, with the exception of a concrete circle in the center where the statue stood.

The architect's drawing described the large circle covering the park by the equation $x^2 + y^2 - 6x - 4y = 612$ and the small circle of concrete by the equation $x^2 + y^2 - 6x - 4y = 87$.

The circles are concentric and measured in feet.

Based on the above information, answer the following questions.

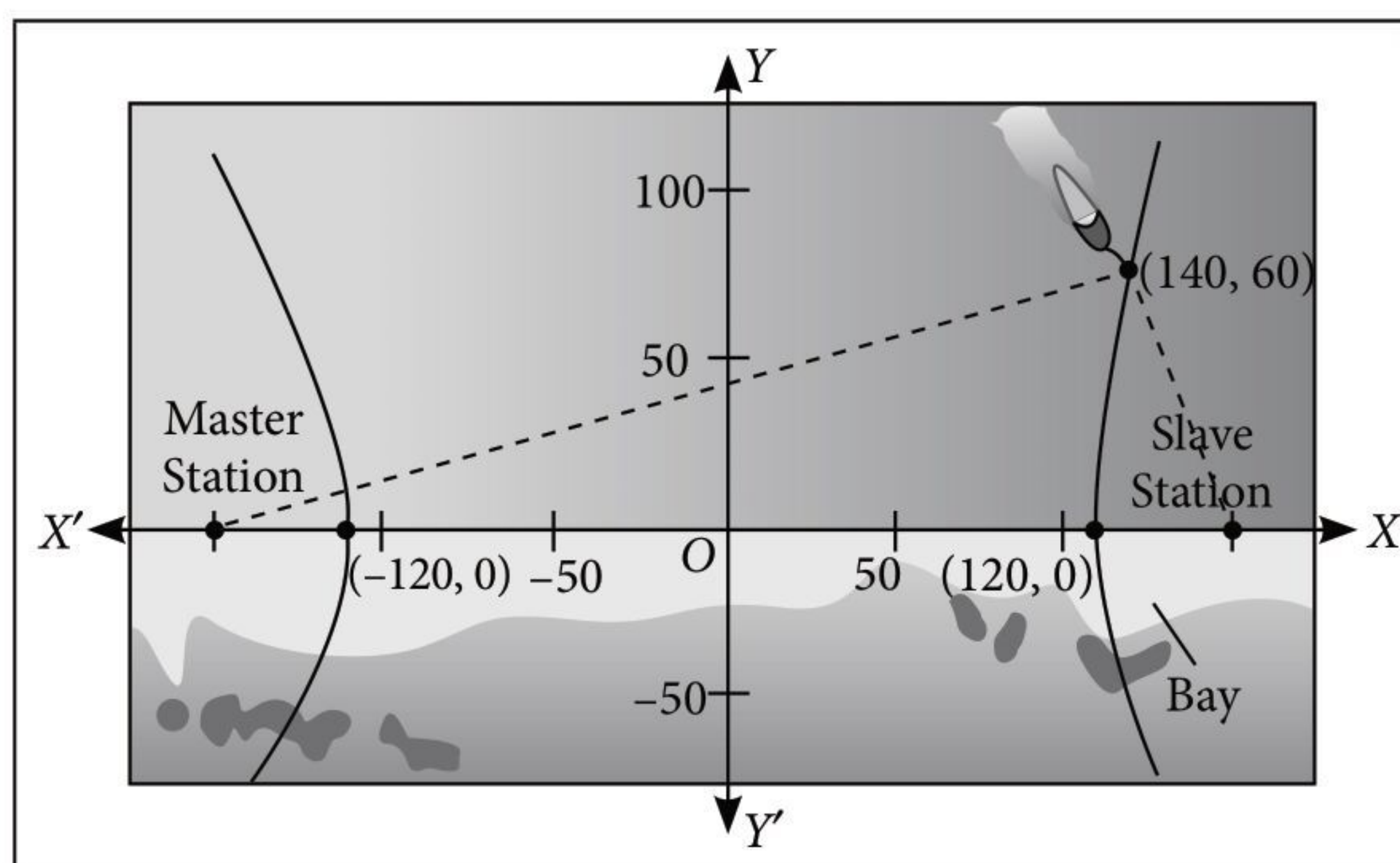


- (i) Find the centre of the circle.
 (a) (3, -2) (b) (-3, -2)
 (c) (3, 2) (d) (3, 0)
- (ii) Find the radii of both circles.
 (a) 525π sq. units (b) 525 sq. units
 (c) 520 sq. units (d) 520π sq. units.
- (iii) Find ratio of radius of larger circle to radius of smaller circle. Also, find the area of the grass.

OR

If an another circle of centre (18, 10) touching the smaller circle, then find the equation of another circle.

38. **Case study 3:** Long distance navigation (LORAM) is a radio navigation system that was developed during second world war. The system allows a pilot to guide aircraft by keeping a constant difference between the aircraft's distance from two fixed points: the master station and the slave station.



Based on the above information, answer the following questions.

- (i) Write the equation for the hyperbola.
 (ii) Find the coordinates of its foci.

SOLUTIONS

1. (a) : $3x^2 + 3y^2 + \lambda xy + 9x + (\lambda - 6)y + 3 = 0$ is an equation of circle if coefficient of xy i.e., $\lambda = 0$

$$\therefore x^2 + y^2 + \frac{9}{3}x - \frac{6y}{3} + \frac{3}{3} = 0$$

$$\text{or } x^2 + y^2 + 3x - 2y + 1 = 0$$

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$

$$g = \frac{3}{2}, f = -1 \text{ and } c = 1$$

$$\therefore \text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{4} + 1 - 1} = \frac{3}{2} \text{ units.}$$

2. (c) : The centre of the circle $(x - a)(x - c) + (y - b)(y - d) = 0$ is mid point of (a, b) and (c, d) .
 i.e., $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

3. (a) : Since $(x, 3)$ and $(3, 5)$ are the extremities of a diameter of a circle with centre $(2, y)$.

$$\therefore 2 = \frac{x+3}{2} \Rightarrow x = 1 \text{ and } y = \frac{3+5}{2} \Rightarrow y = 4$$

$$\Rightarrow x = 1 \text{ and } y = 4.$$

4. (c) : The given equation is $x^2 + y^2 - 2px + 4y - 12 = 0$
 Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$,
 we get $g = -p, f = 2, c = -12$

$$\therefore \text{Radius} = \sqrt{(-p)^2 + 2^2 + 12} \Rightarrow 5 = \sqrt{p^2 + 4 + 12}$$

$$\Rightarrow 25 - 16 = p^2 \Rightarrow p^2 = 9 \Rightarrow p = \pm 3$$

5. (a) : We have given, focus of parabola is $S(0, -3)$, i.e., $(0, -a)$ lies on the y -axis and equation of directrix is $y = 3$ i.e., $y = a$.

Therefore, y -axis is the axis of the parabola. The required equation of the parabola is

$$x^2 = -4ay \Rightarrow x^2 = -4 \times 3y \Rightarrow x^2 = -12y.$$

6. (b) : Since the given equation of parabola is $y^2 = 4ax$ and the parabola passes through the point $(3, 2)$.

$$\therefore 4 = 4a(3) \Rightarrow a = 1/3$$

$$\therefore \text{Length of latus rectum} = 4a = 4/3$$

7. (d) : Compare the given equation of the parabola $y^2 = 12x$ with the standard equation $y^2 = 4ax$.

$$\Rightarrow 4a = 12 \Rightarrow a = 3 \therefore \text{Focus} = (3, 0)$$

8. (d) : We have, $(x - 1)^2 = 2(y - 2)$... (i)

Put $x = X + 1$ and $y = Y + 2$

$$\therefore \text{(i) can be written as } X^2 = 2Y$$

$$\therefore 4a = 2 \Rightarrow a = \frac{1}{2}$$

Equation of directrix of parabola $X^2 = 4aY$ is $Y = -a$.

$$\Rightarrow Y = -\frac{1}{2} \therefore y - 2 = -\frac{1}{2}$$

$$\Rightarrow 2y - 4 = -1 \Rightarrow 2y - 3 = 0$$

9. (b) : Given equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$

$$\text{As we know, } e = \sqrt{1 - \frac{a^2}{b^2}} \Rightarrow e^2 = \frac{(b^2 - a^2)}{b^2}$$

$$\Rightarrow b^2 e^2 = b^2 - a^2 \Rightarrow a^2 = b^2(1 - e^2)$$

10. (b) : Given equation of ellipse is $12x^2 + 7y^2 = 84$

$$\text{or } \frac{x^2}{7} + \frac{y^2}{12} = 1$$

The denominator of x^2 is less than the denominator of y^2 . Therefore the standard equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b.$$

$$\therefore b^2 = 12, a^2 = 7$$

$$\text{As we know, } e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{7}{12}} = \sqrt{\frac{5}{12}}$$

11. (d) : Given equation of ellipse is

$$3x^2 + y^2 = 12 \Rightarrow \frac{x^2}{4} + \frac{y^2}{12} = 1$$

Clearly, $12 > 4$

The denominator of x^2 is less than the denominator of y^2 . Therefore the standard equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b.$$

$$\therefore b^2 = 12 \Rightarrow b = 2\sqrt{3} \text{ and } a^2 = 4$$

$$\therefore \text{Length of latus rectum} = \frac{2a^2}{b} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$$

12. (a) : Coordinates of foci are $(\pm ae, 0)$. So, $ae = 4$

$$\Rightarrow a \times \frac{1}{3} = 4 \quad \left(\because e = \frac{1}{3} \right) \therefore a = 12$$

$$b^2 = a^2(1 - e^2) \Rightarrow b^2 = (12)^2 \left(1 - \frac{1}{9} \right) = 144 \times \frac{8}{9} = 128$$

Thus, the required equation of ellipse is

$$\frac{x^2}{144} + \frac{y^2}{128} = 1 \text{ or } \frac{x^2}{9} + \frac{y^2}{8} = 16$$

13. (a) : According to the question,

$$2ae = 16 \Rightarrow ae = 8$$

$$\text{and } e = \sqrt{2}$$

$$\text{From (i) and (ii), } \sqrt{2}a = 8 \Rightarrow a = 4\sqrt{2}$$

Now, $b^2 = a^2(e^2 - 1)$, for hyperbola

$$\Rightarrow b^2 = 32(2 - 1) \Rightarrow b^2 = 32$$

So, the required equation of the hyperbola is

$$\frac{x^2}{32} - \frac{y^2}{32} = 1 \text{ or } x^2 - y^2 = 32$$

14. (b) : The given hyperbola can be written as

$$\frac{x^2}{1} - \frac{y^2}{1/4} = 1 \therefore a^2 = 1, b^2 = \frac{1}{4}$$

$$\text{Now, } b^2 = a^2(e^2 - 1)$$

$$\Rightarrow \frac{1}{4} = 1(e^2 - 1) \Rightarrow e^2 = \frac{5}{4} \Rightarrow e = \frac{\sqrt{5}}{2}$$

15. (b) : Let us write the given equation in the form $(x^2 - 2x) + (y^2 + 4y) = 8$

Now, completing the squares, we get

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 1 + 4$$

$$\text{or } (x - 1)^2 + (y + 2)^2 = 13$$

Comparing it with the standard form of the equation of the circle, we see that the centre of the circle is $(1, -2)$ and radius is $\sqrt{13}$.

16. (a) : Let the equation of the parabola with vertex at origin and axis along x -axis is $y^2 = 4ax$... (i)

As, (i) passes through $(2, 8)$.

$$\therefore (8)^2 = 4a(2) \Rightarrow a = 8$$

Putting value of a in (i), we get required equation of parabola $y^2 = 32x$.

17. (d) : Given, $\frac{x^2}{25/3} - \frac{y^2}{25/3} = 1 \Rightarrow a^2 = \frac{25}{3}, b^2 = \frac{25}{3}$

$$\therefore b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{3} = \frac{25}{3}(e^2 - 1) \Rightarrow e^2 = 2.$$

18. (b) : Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of ellipse.

It is given that it passes through $(-3, 1)$ and $(2, -2)$.

$$\therefore \frac{9}{a^2} + \frac{1}{b^2} = 1 \text{ ... (i), and } \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4} \text{ ... (ii)}$$

$$\text{Solving (i) and (ii) we get } a^2 = \frac{32}{3} \text{ and } b^2 = \frac{32}{5}$$

Hence, the required equation of ellipse is $3x^2 + 5y^2 = 32$.

19. (d) : Clearly, Reason is true.

$$\text{Now, } 5x^2 + 9y^2 - 54y + 36 = 0$$

$$\Rightarrow 5x^2 + 9(y - 3)^2 = 45 \Rightarrow \frac{x^2}{3^2} + \frac{(y - 3)^2}{(\sqrt{5})^2} = 1$$

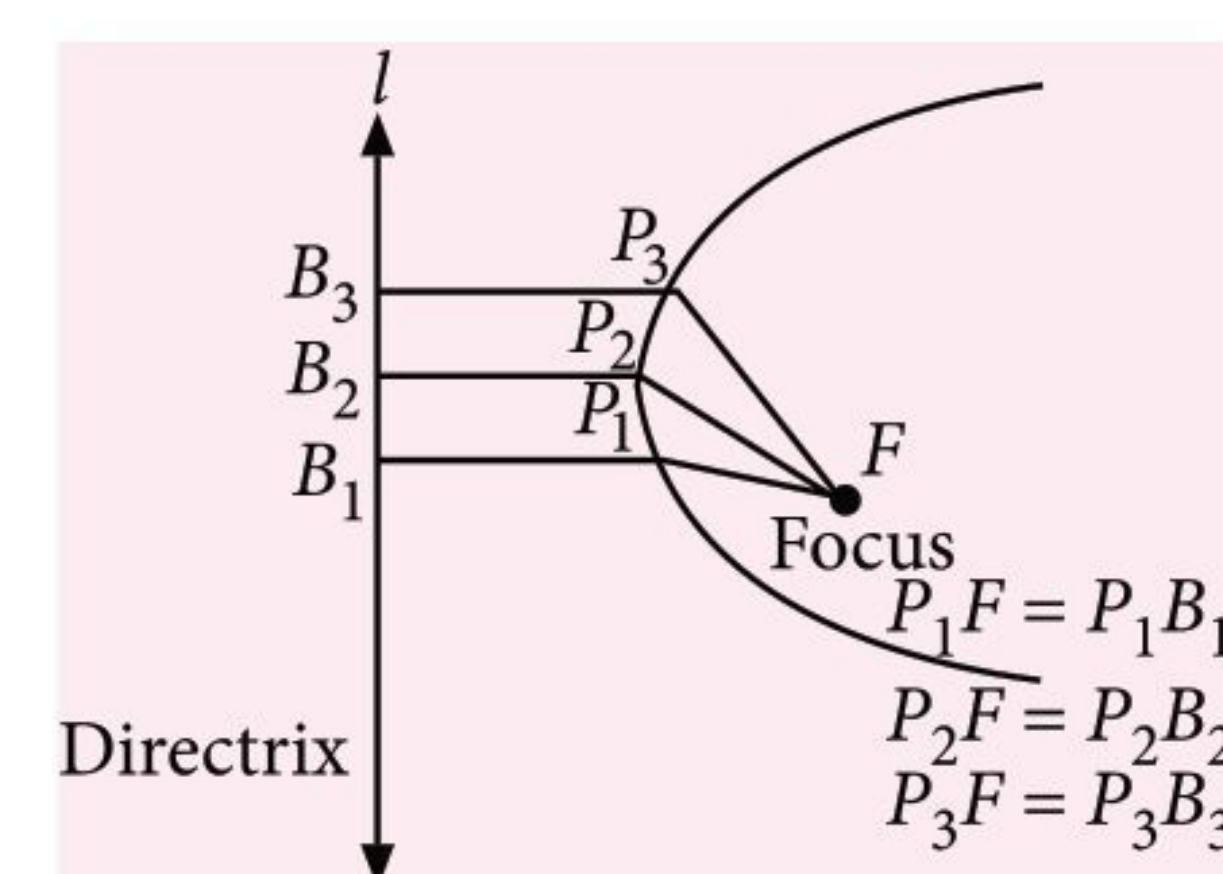
$$\therefore \text{Length of major axis} = 2 \times 3 = 6$$

$$\text{and length of minor axis} = 2 \times \sqrt{5} = 2\sqrt{5}$$

$\therefore$ A is false but R is true.

20. (b) : The fixed line is called the directrix of the parabola. The fixed point is called the focus.

$\therefore$ A and R both are true but R is not the correct explanation of A.



$$\text{21. The given lines } 3x - 4y + 4 = 0 \text{ \& } 2\left(3x - 4y - \frac{7}{2}\right) = 0$$

are parallel to each other, and distance between given parallel lines gives the diameter of the circle.

$$\therefore \text{Diameter} = \left| \frac{4 + \frac{7}{2}}{\sqrt{9+16}} \right| = \left(\frac{15}{2} \right) \times \frac{1}{5} = \frac{3}{2}$$

$\therefore$ Radius of circle = $3/4$.

OR

Let a be the radius of the circle, then $(-a, -a)$ will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.

$$\text{Since } d = \left| \frac{-3a + 4(a) + 8}{\sqrt{9+16}} \right| = \left| \frac{a+8}{5} \right| = \pm \left(\frac{a+8}{5} \right)$$

$$\therefore a = \frac{a+8}{5} \text{ and } a = \frac{-a-8}{5} \Rightarrow a = 2 \text{ and } a = -4/3$$

But $a \neq -4/3 \Rightarrow a = 2$.

Hence, required equation of circle is $(x+2)^2 + (y+2)^2 = 2^2$
 $\Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$

22. Given equation of the circle is

$$x^2 + y^2 - 4x - 6y + 11 = 0 \quad \dots(i)$$

Compare (i) with general equation

$x^2 + y^2 + 2gx + 2fy + c = 0$ of circle, we get

$$2g = -4 \text{ and } 2f = -6 \Rightarrow \text{Centre} = (-g, -f) = (2, 3)$$

Let the co-ordinates of the other end of diameter of the circle are (x, y) . Now, $(2, 3)$ is the mid-point of diameter.

$$\therefore 2 = \frac{3+x}{2} \Rightarrow x = 1 \text{ \& } 3 = \frac{4+y}{2} \Rightarrow y = 2$$

Hence, co-ordinates of the other end of the diameter are $(1, 2)$.

23. Since, vertices of the ellipse are $(\pm 13, 0)$, and foci are $(\pm 5, 0)$.

$\therefore$ Major axis is along x -axis.

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$
 $\dots(i)$

$$\text{Foci} = (\pm 5, 0) = (\pm c, 0) \Rightarrow c = 5 \Rightarrow c^2 = 25.$$

$$\Rightarrow a^2 - b^2 = 25 \quad \dots(ii)$$

$$\text{Vertices} = (\pm 13, 0) = (\pm a, 0) \Rightarrow a = 13 \quad \dots(iii)$$

$$\text{From (ii) and (iii), } b^2 = 169 - 25 = 144 \Rightarrow b = 12 \quad \dots(iv)$$

Putting $a = 13, b = 12$ in (i), we get $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

OR

Given, $e = 5/8$

Consider the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$
 $a > b$

Since foci of this ellipse are $(\pm ae, 0)$.

$\therefore$ Distance between foci = $2ae = 10$

$$\Rightarrow a = \frac{10}{2e} = \frac{10 \times 8}{2 \times 5} = 8$$

As we know, $b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 64 \left(1 - \frac{25}{64} \right) \Rightarrow b^2 = 39$$

$\therefore$ Length of latus rectum of ellipse

$$= \frac{2b^2}{a} = 2 \left(\frac{39}{8} \right) = \frac{39}{4}$$

24. Given, equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Major axis is along x -axis as $25 > 9$.

Here, $a^2 = 25, b^2 = 9$.

$$\therefore \text{Eccentricity } (e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

25. Given, $4x^2 + y^2 = 100 \Rightarrow \frac{x^2}{25} + \frac{y^2}{100} = 1$
 Here, $a^2 = 25, b^2 = 100$

$$\text{Now, } e = \sqrt{1 - \frac{a^2}{b^2}} \quad (\because a < b)$$

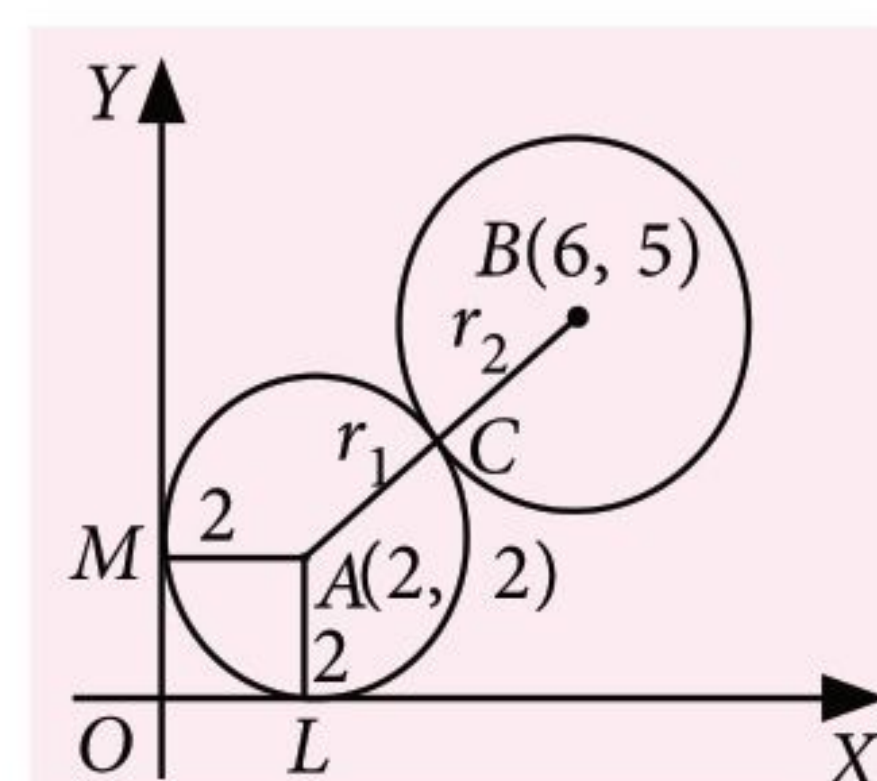
$$= \sqrt{1 - \frac{25}{100}} = \frac{\sqrt{75}}{10} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

26. Given, $AC = 2$ units

Also $A \equiv (2, 2)$, and $B \equiv (6, 5)$

$$\therefore AB = \sqrt{(2-6)^2 + (2-5)^2} = 5 \text{ units}$$

$$\therefore BC = AB - AC = 5 - 2 = 3 \text{ units}$$



Hence, the equation of the required circle whose centre is $B(6, 5)$ and radius is 3 units, is $(x-6)^2 + (y-5)^2 = 3^2$
 or $x^2 + y^2 - 12x - 10y + 52 = 0$.

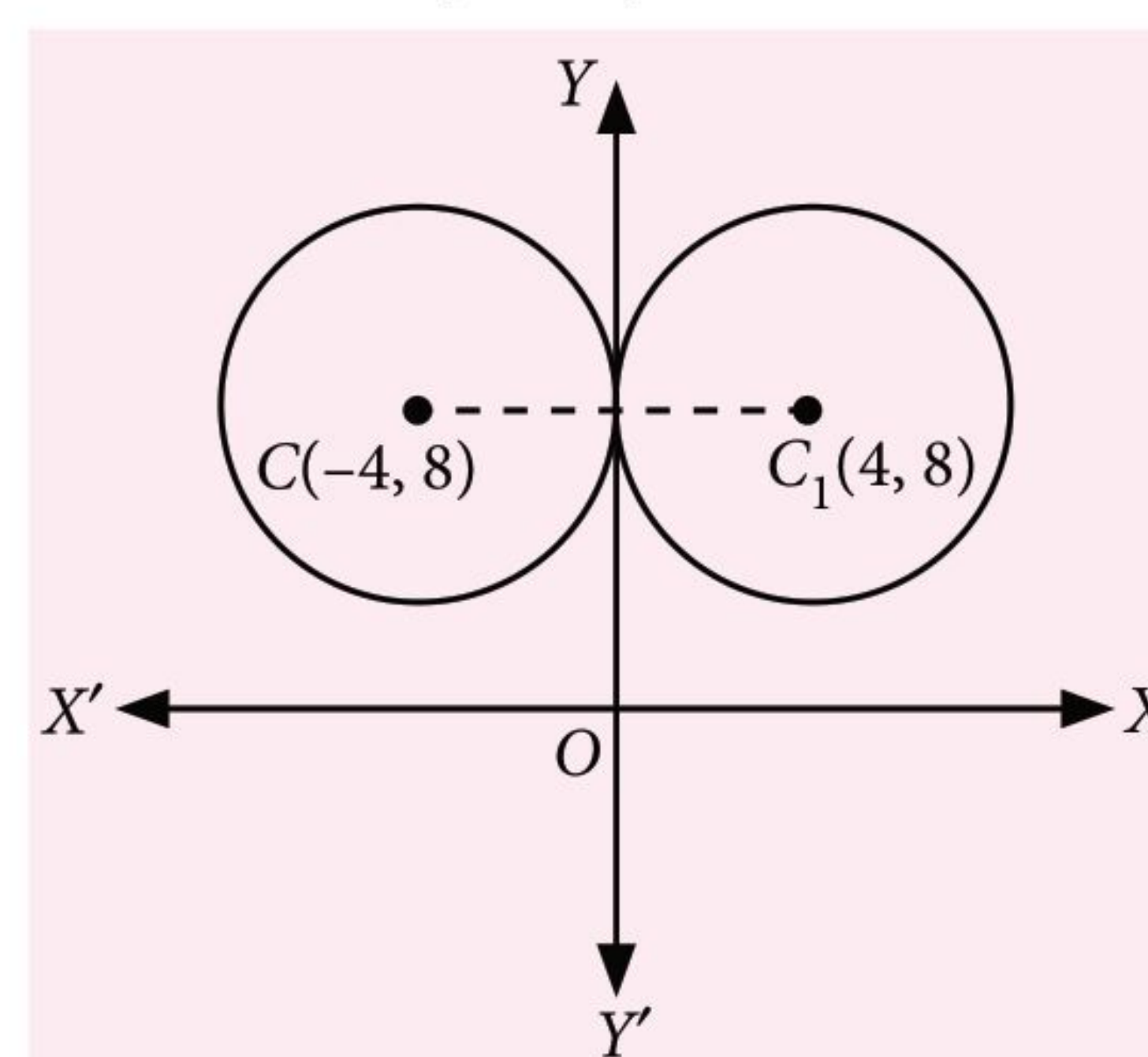
OR

The equation of the given circle is

$$x^2 + y^2 + 8x - 16y + 64 = 0$$

$$\Rightarrow (x+4)^2 + (y-8)^2 = 4^2 \Rightarrow (x-(-4))^2 + (y-8)^2 = 4^2$$

Clearly, its centre is at $(-4, 8)$ and radius = 4 units



The image of this circle in the line mirror $x = 0$ has its centre $C_1(4, 8)$ and radius 4 units. So, its equation is $(x - 4)^2 + (y - 8)^2 = 4^2$ or, $x^2 + y^2 - 8x - 16y + 64 = 0$.

27. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is $C_1(3, -2)$. Therefore, the required circle passes through the point $C_1(3, -2)$ and has its centre at the point $C(4, 5)$. So, its radius is given by

$$CC_1 = \sqrt{(4-3)^2 + (5+2)^2} = \sqrt{50} \text{ units,}$$

Hence, the equation of the required circle is

$$(x-4)^2 + (y-5)^2 = (\sqrt{50})^2 \text{ or } x^2 + y^2 - 8x - 10y - 9 = 0$$

OR

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let e be its eccentricity.

It is given that the coordinates of foci are $(\pm 5, 0)$ and the equation of one of the directrices is $x = 36/5$.

$$\therefore ae = 5 \text{ and } \frac{a}{e} = \frac{36}{5} \Rightarrow ae \times \frac{a}{e} = 5 \times \frac{36}{5}$$

$$\Rightarrow a^2 = 36 \Rightarrow a = 6$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = a^2 - (ae)^2 = 36 - 25 = 11 \Rightarrow b = \sqrt{11}$$

Substituting $a = 6$ and $b = \sqrt{11}$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we

obtain $\frac{x^2}{36} + \frac{y^2}{11} = 1$ as the required equation of ellipse.

28. Let S and S' be two foci of the required ellipse. Then, the coordinates of S and S' are $(2, 3)$ and $(-2, 3)$ respectively.

$$\therefore SS' = 4$$

Let $2a$ and $2b$ be the lengths of the major and minor axes of the ellipse respectively and e be its eccentricity.

$$\text{Then, } SS' = 2ae \Rightarrow 2ae = 4 \Rightarrow ae = 2.$$

$$\text{Now, } b^2 = a^2(1 - e^2) \Rightarrow 5 = a^2 - 2^2 \Rightarrow a = 3.$$

Let $P(x, y)$ be any point on the ellipse. Then,

$$SP + S'P = 2a$$

$$\Rightarrow \sqrt{(x-2)^2 + (y-3)^2} + \sqrt{(x+2)^2 + (y-3)^2} = 6$$

$$\Rightarrow [(x-2)^2 + (y-3)^2] - [(x+2)^2 + (y-3)^2]$$

$$= 36 - 12 \left\{ \sqrt{(x+2)^2 + (y-3)^2} \right\}$$

$$\Rightarrow -8x = 36 - 12 \left\{ \sqrt{(x+2)^2 + (y-3)^2} \right\}$$

$$\Rightarrow (2x+9)^2 = 9 \{(x+2)^2 + (y-3)^2\}$$

$$\Rightarrow 4x^2 + 81 + 36x = 9(x^2 + 4 + 4x + y^2 + 9 - 6y)$$

$$\Rightarrow 5x^2 + 9y^2 - 54y + 36 = 0 \text{ is the required equation of the ellipse.}$$

29. Given, $e = 2/3$ and length of latus rectum = 5

$$\Rightarrow \frac{2b^2}{a} = 5 \Rightarrow b^2 = \frac{5a}{2} \quad \dots (i)$$

As we know, $b^2 = a^2(1 - e^2)$,

$$\Rightarrow \frac{5a}{2} = a^2 \left(1 - \frac{4}{9} \right) \Rightarrow \frac{5}{2} = \frac{5a}{9} \Rightarrow a = \frac{9}{2} \Rightarrow a^2 = \frac{81}{4}$$

$$\text{and } b^2 = \frac{5 \times 9}{2 \times 2} = \frac{45}{4} \quad (\text{From (i)})$$

Hence, the required equation of the ellipse is

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1 \text{ or } \frac{4x^2}{45} + \frac{4y^2}{81} = 1$$

30. The equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$
 $\therefore a^2 = 25$ and $b^2 = 9$

$$\text{Eccentricity of ellipse} = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

So, the coordinates of foci are $(\pm 4, 0)$

Since the coordinates of foci of hyperbola coincide with the foci of the ellipse. So the coordinates of foci of hyperbola are $(\pm 4, 0)$.

Let equation of required hyperbola be $\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1$

Let e' be the eccentricity of required hyperbola,

$$\text{then } a'e' = 4 \Rightarrow 2a' = 4 \quad (\because e' = 2)$$

$$\Rightarrow a' = 2 \therefore b'^2 = a'^2(e'^2 - 1) \Rightarrow b'^2 = 4(4 - 1) = 12$$

Thus, required equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$.

31. Consider the point be $P(x, y)$.

$$\text{Now, distance of } P \text{ from } (4, 0) = \sqrt{(x-4)^2 + y^2}$$

$$\text{and the distance of } P \text{ from } (-4, 0) = \sqrt{(x+4)^2 + y^2}$$

According to the question,

$$\sqrt{(x+4)^2 + y^2} - \sqrt{(x-4)^2 + y^2} = 2$$

$$\Rightarrow \sqrt{(x+4)^2 + y^2} = 2 + \sqrt{(x-4)^2 + y^2}$$

On squaring both sides, we get

$$x^2 + 8x + 16 + y^2 = 4 + x^2 - 8x + 16 + y^2 + 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow 16x - 4 = 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow 4(4x - 1) = 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow 16x^2 - 8x + 1 = x^2 + 16 - 8x + y^2$$

$$\Rightarrow 15x^2 - y^2 = 15, \text{ which represents a hyperbola.}$$

OR

Let $P(\alpha, \beta)$ be any point on hyperbola

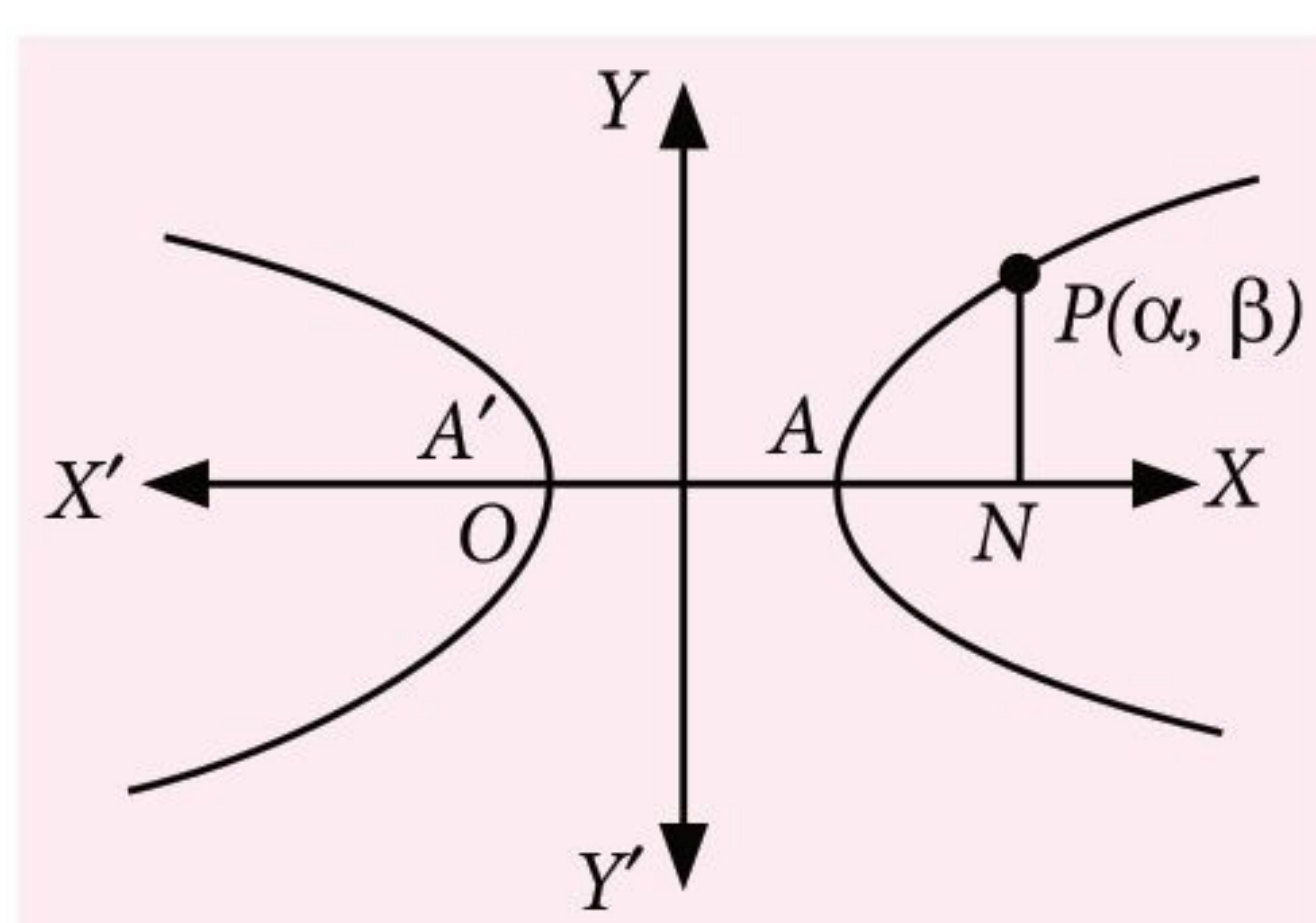
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Then, } \frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$$

$$\Rightarrow \frac{\alpha^2}{a^2} - 1 = \frac{\beta^2}{b^2}$$

$$\Rightarrow \frac{\alpha^2 - a^2}{a^2} = \frac{\beta^2}{b^2} \Rightarrow \frac{(\alpha - a)(\alpha + a)}{a^2} = \frac{\beta^2}{b^2}$$

$$\Rightarrow \frac{(AN)(A'N)}{a^2} = \frac{(PN)^2}{b^2} \Rightarrow \frac{(PN)^2}{(AN)(A'N)} = \frac{b^2}{a^2}$$



32. The given equation is $y^2 = 4x + 4$

$$\Rightarrow y^2 - 4y = 4x \Rightarrow y^2 - 4y + 4 = 4x + 4$$

$$\Rightarrow (y - 2)^2 = 4(x + 1) \quad \dots(i)$$

Shifting the origin to the point $(-1, 2)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y , we have

$$x = X + (-1), y = Y + 2 \quad \dots(ii)$$

Using these relations, (i) reduces to $Y^2 = 4X$

This is of the form $Y^2 = 4aX$.

On comparing we get $4a = 4 \Rightarrow a = 1$.

Vertex: The coordinates of the vertex w.r.t. new axes are $(X = 0, Y = 0)$.

The coordinates of the vertex w.r.t. old axes are $(-1, 2)$
[Putting $X = 0, Y = 0$ in (ii)]

Focus: The coordinates of the focus w.r.t. new axes are $(1, 0)$.

So, coordinates of the focus w.r.t. old axes are $(0, 2)$
[Putting $X = 1, Y = 0$ in (ii)]

Directrix: Equation of the directrix of the parabola w.r.t. new axes is $X = -1$

So, equation of the directrix of the parabola w.r.t. old axes is $x = -2$.

Axis : Equation of axis of the parabola w.r.t. new axes is $Y = 0$.

So, equation of axis w.r.t. old axes is $y = 2$
[Putting $Y = 0$ in (ii)]

Latus rectum: The length of the latus rectum = 4.

33. The given equation of the ellipse is

$$x^2 + 4y^2 + 8y - 2x + 1 = 0$$

$$\Rightarrow x^2 - 2x + 4y^2 + 8y = -1$$

$$\Rightarrow (x^2 - 2x + 1) + 4(y^2 + 2y + 1) = -1 + 1 + 4$$

$$\Rightarrow (x - 1)^2 + 4(y + 1)^2 = 4$$

$$\Rightarrow \frac{(x - 1)^2}{2^2} + \frac{(y + 1)^2}{1^2} = 1 \quad \dots(i)$$

Shifting the origin to $(1, -1)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y we obtain

$$x = X + 1, y = Y - 1 \quad \dots(ii)$$

Using these relations, (i) reduces to $\frac{X^2}{2^2} + \frac{Y^2}{1^2} = 1$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

On comparing, we get

$$a^2 = 2^2 \text{ and } b^2 = 1 \Rightarrow a = 2 \text{ and } b = 1.$$

Let e be the eccentricity of the ellipse.

$$\text{Then, } b^2 = a^2(1 - e^2) \Rightarrow 1 = 4(1 - e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$$

The coordinates of foci with respect to new axes are $(\pm ae, 0)$ i.e., $(\pm\sqrt{3}, 0)$.

So coordinates of foci with respect to old axes are $(1 \pm \sqrt{3}, -1)$ [Putting $X = \pm\sqrt{3}, Y = 0$ in (ii)]

$$\text{Length of the latus-rectum} = \frac{2b^2}{a} = \frac{2(1)^2}{2} = 1.$$

OR

$$\text{Given, } \frac{x^2}{169} + \frac{y^2}{144} = 1 \text{ where } a = 13, b = 12.$$

$$\therefore \text{Length of major axis} = 2a = 26 \text{ units}$$

$$\text{Length of minor axis} = 2b = 24 \text{ units.}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2(144)}{13} = 22.15 \text{ units}$$

$$\text{Vertices} \cong (\pm a, 0) \cong (\pm 13, 0)$$

$$\text{Foci} \cong (\pm\sqrt{a^2 - b^2}, 0) = (\pm\sqrt{169 - 144}, 0) = (\pm 5, 0)$$

$$\text{Eccentricity, } e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\text{Equation of directrix, } x = \pm \frac{a}{e} = \pm \frac{13 \times 13}{5} = \pm \frac{169}{5}.$$

34. The given equation is $4y^2 + 12x - 12y + 39 = 0$

$$\Rightarrow 4\left(y^2 - 3y + \frac{9}{4}\right) = -12x - 39 + 9$$

$$\Rightarrow 4\left(y - \frac{3}{2}\right)^2 = -12\left(x + \frac{5}{2}\right)$$

$$\Rightarrow \left(y - \frac{3}{2}\right)^2 = -3\left(x + \frac{5}{2}\right) \quad \dots(i)$$

Shifting the origin to the point $(-5/2, 3/2)$ without rotating the axes and denoting the new coordinates w.r.t. these axes by X and Y , we have

$$x = X + \left(-\frac{5}{2}\right), y = Y + \frac{3}{2} \quad \dots(ii)$$

Using these relations, (i) reduces to

$$Y^2 = -3X$$

This is of the form $Y^2 = -4aX$.

On comparing, we get $a = \frac{3}{4}$

Vertex: The coordinates of the vertex w.r.t. new axes are $(X = 0, Y = 0)$

So, coordinates of the vertex w.r.t. old axes are $\left(-\frac{5}{2}, \frac{3}{2}\right)$

[Putting $X = 0, Y = 0$ in (ii)]

Focus: The coordinates of the focus of the parabola w.r.t. new axes are $\left(-\frac{3}{4}, 0\right)$

So, coordinates of the focus w.r.t. old axes are

$$\left(-\frac{13}{4}, \frac{3}{2}\right) \quad \left[\text{Putting } X = -\frac{3}{4}, Y = 0 \text{ in (ii)} \right]$$

Directrix: The equation of the directrix of the parabola w.r.t. new axes is $X = \frac{3}{4}$.

So, equation of the directrix of the parabola w.r.t. old axes is $x = -\frac{7}{4}$ [Putting $X = 3/4$ in (ii)]

OR

In a parabola, vertex is the mid-point of the focus and the point of intersection of the axis and directrix. So, let (x_1, y_1) be the coordinates of the point of intersection of the axis and directrix. Then, $(2, -3)$ is the mid-point of the line segment joining $(0, 5)$ and (x_1, y_1) .

$$\therefore \frac{x_1 + 0}{2} = 2 \text{ and } \frac{y_1 + 5}{2} = -3 \Rightarrow x_1 = 4, y_1 = -11.$$

Thus, the directrix meets the axis at $(4, -11)$.

Let A be the vertex and S be the focus of the required parabola.

$$\text{Then, } m_1 = \text{slope of } AS = \frac{5+3}{0-2} = -4$$

Let m_2 be the slope of the directrix.

$$\text{Then, } m_1 m_2 = -1 \Rightarrow m_2 = \frac{-1}{m_1} = \frac{1}{4}.$$

Thus, the directrix passes through $(4, -11)$ and has slope $\frac{1}{4}$. Therefore, the equation of the directrix is

$$y + 11 = \frac{1}{4}(x - 4) \Rightarrow x - 4y - 48 = 0$$

Let $P(x, y)$ be any point on the required parabola, and let PM be the length of the perpendicular from P on the directrix. Then,

$$SP = PM \Rightarrow SP^2 = PM^2$$

$$\Rightarrow (x-0)^2 + (y-5)^2 = \left(\frac{x-4y-48}{\sqrt{1^2 + (-4)^2}} \right)^2$$

$$\Rightarrow 17x^2 + 17y^2 - 170y + 425 = x^2 + 16y^2 + 2304 - 8xy - 96x + 384y$$

$\Rightarrow 16x^2 + y^2 + 8xy + 96x - 554y - 1879 = 0$ is the required equation of parabola.

35. We have, $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$$\Rightarrow 9(x^2 - 2x) - 16(y^2 - 2y) = 151$$

$$\Rightarrow 9(x^2 - 2x + 1) - 16(y^2 - 2y + 1) = 144$$

$$\Rightarrow 9(x-1)^2 - 16(y-1)^2 = 144$$

$$\Rightarrow \frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1 \quad \dots(i)$$

Shifting the origin at $(1, 1)$ without rotating the axes and denoting the new coordinates with respect to these axes by X and Y , we have

$$x = X + 1 \text{ and } y = Y + 1 \quad \dots(ii)$$

Using these relations, (i) reduces to

$$\frac{X^2}{16} - \frac{Y^2}{9} = 1 \text{ where } a^2 = 16 \text{ and } b^2 = 9.$$

Centre : The coordinates of the centre with respect to the new axes are $(0, 0)$.

So, the coordinates of the centre with respect to the old axes are $(1, 1)$ [Putting $X = 0, Y = 0$ in (ii)]

Length of the transverse axis $= 2a = 8$

Length of the conjugate axis $= 2b = 6$

36. (i) We have given, the length of major axis is 10 cm and length of minor axis is 8 cm.

$$\therefore 2a = 10$$

$$\Rightarrow a = 5 \text{ cm}$$

$$\text{And } 2b = 8 \text{ cm.}$$

$$\Rightarrow b = 4 \text{ cm}$$

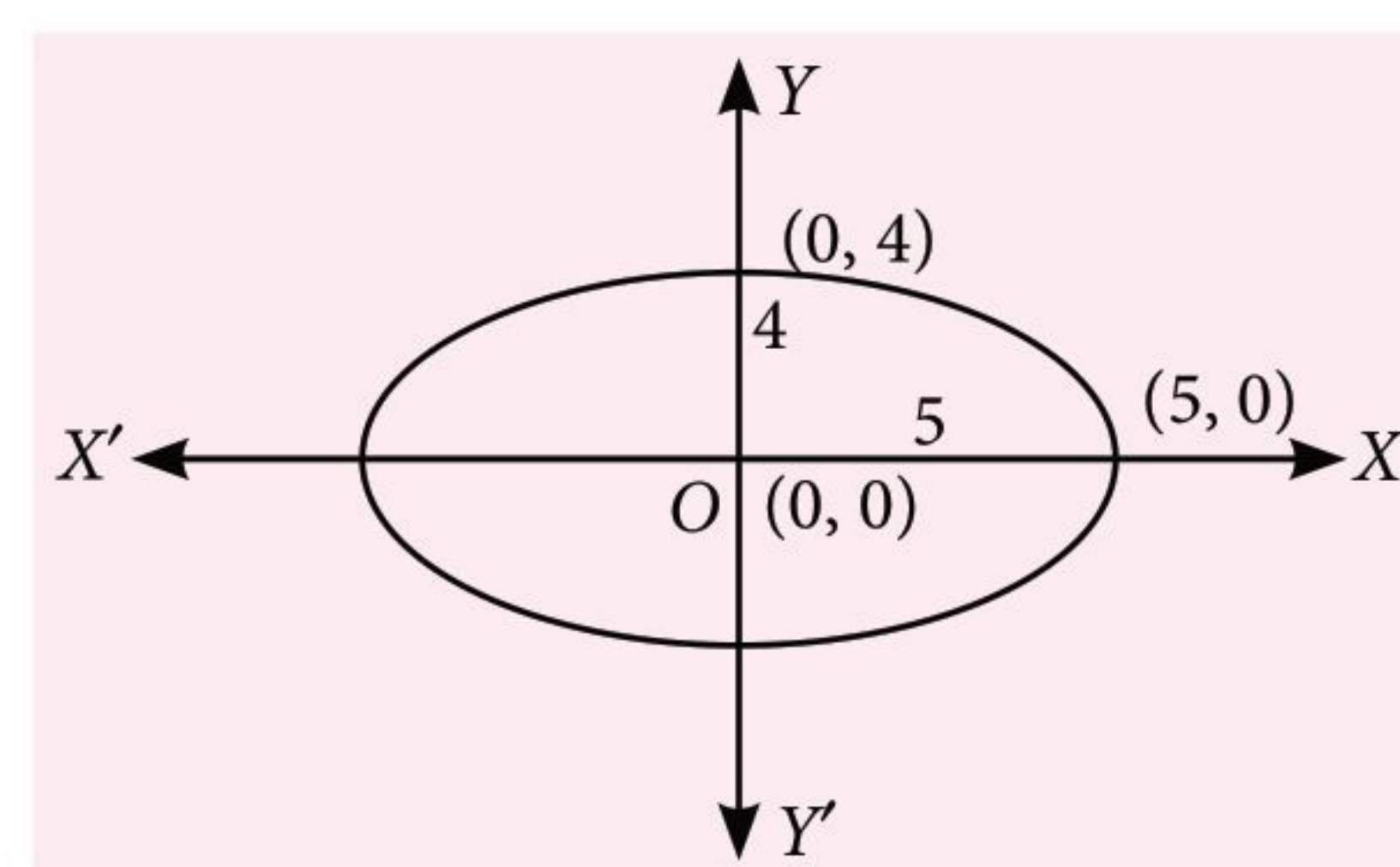
Consider the equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \text{The required equation of ellipse is } \frac{x^2}{25} + \frac{y^2}{16} = 1.$$

$$\text{The ends of latus rectum is } \left(\pm ae, \pm \frac{b^2}{a} \right)$$

$$\therefore \left(\pm ae, \pm \frac{b^2}{a} \right) \equiv \left(\pm \frac{3}{5} \times 5, \pm \frac{16}{5} \right) \equiv \left(\pm 3, \pm \frac{16}{5} \right)$$



(ii) The distance between foci is given by $2ae$.

$$\therefore 2ae = 2 \times 5 \times \frac{3}{5} = 6.$$

(iii) The eccentricity of ellipse is $e = \sqrt{1 - \frac{b^2}{a^2}}$.

$$\therefore e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

OR

The equation of directrix is $x = \pm \frac{a}{e}$

$$\therefore x = \pm \frac{5}{1} \times \frac{5}{3} = \pm \frac{25}{3}$$

37. (i) We have, $C_1: x^2 + y^2 - 6x - 4y = 612$

Comparing the given equation with general equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, we have

$$g = -3, f = -2, c = -612$$

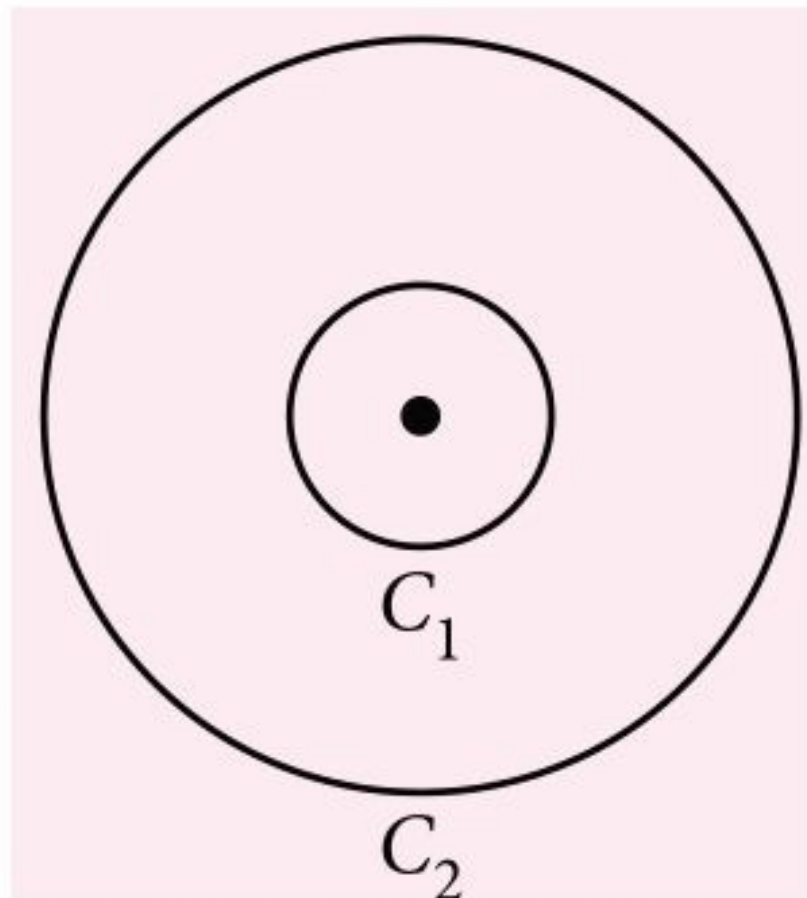
$\therefore$ Centre is $(-g, -f)$ i.e., $(3, 2)$

(ii) Radius is $\sqrt{g^2 + f^2 - c}$

$$\text{i.e., } \sqrt{9 + 4 + 612} = \sqrt{625} = 25$$

Similarly, $C_2: x^2 + y^2 - 6x - 4y = 87$

$\therefore$ Centre is $(3, 2)$ and radius is 10.



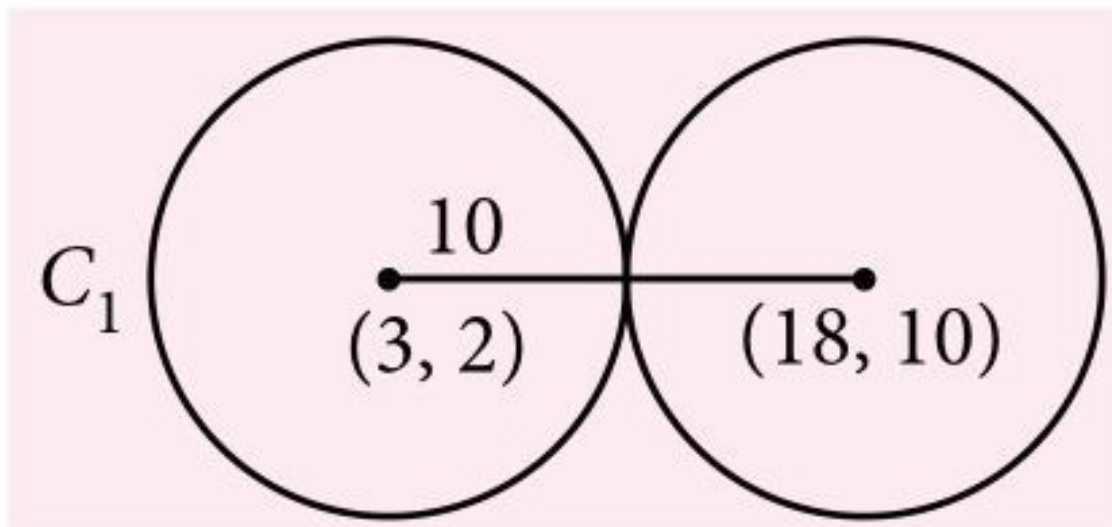
(iii) Required ratio = $\frac{\text{Radius of larger circle}}{\text{Radius of smaller circle}} = \frac{25}{10} = \frac{5}{2}$

The area of the grass is given by $= \pi(25^2 - 10^2)$
 $= \pi(625 - 100) = 525\pi$ sq. units.

OR

The distance between centres is given as

$$\begin{aligned} &= \sqrt{(18-3)^2 + (10-2)^2} \\ &= \sqrt{15^2 + 8^2} = \sqrt{225 + 64} \\ &= \sqrt{289} = 17 \end{aligned}$$



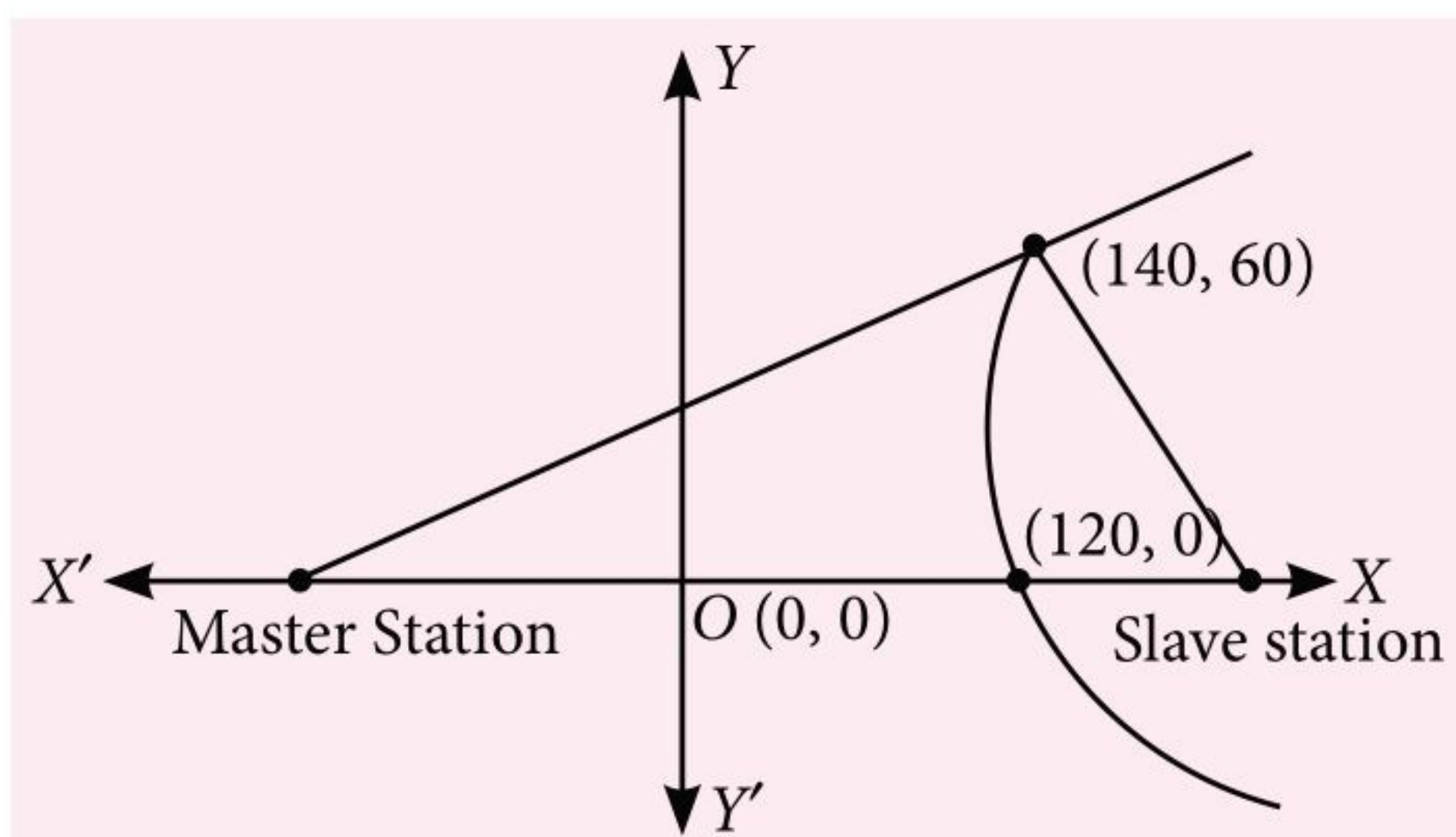
The radius of required circle is $17 - 10 = 7$ units.

$\therefore$ The equation of required circle is

$$(x - 18)^2 + (y - 10)^2 = 7^2$$

$$\text{or } x^2 + y^2 - 36x - 20y + 375 = 0$$

38. (i)



Let the equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The vertex of the hyperbola is $(\pm a, 0)$.

From the diagram, the vertex is $(\pm 120, 0)$

$$\therefore a = 120.$$

Since, the point $(140, 60)$ lies on the hyperbola.

$$\text{Then } \frac{140^2}{120^2} - \frac{60^2}{b^2} = 1 \Rightarrow (140)^2 b^2 - (120)^2 (60)^2 = (120)^2 b^2$$

$$\Rightarrow b^2 = \frac{129600}{13}$$

$\therefore$ The equation of required hyperbola is

$$\Rightarrow \frac{x^2}{(120)^2} - \frac{y^2}{\left(\frac{129600}{13}\right)} = 1 \Rightarrow \frac{x^2}{14400} - \frac{13y^2}{129600} = 1.$$

(ii) The eccentricity of the hyperbola M is

$$e = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow e = \sqrt{1 + \frac{129600}{13 \times 14400}} = \sqrt{1 + \frac{9}{13}} = \sqrt{\frac{22}{13}}$$

The coordinates of foci are $(\pm ae, 0)$ i.e., $\left(\pm 120\sqrt{\frac{22}{13}}, 0\right)$.



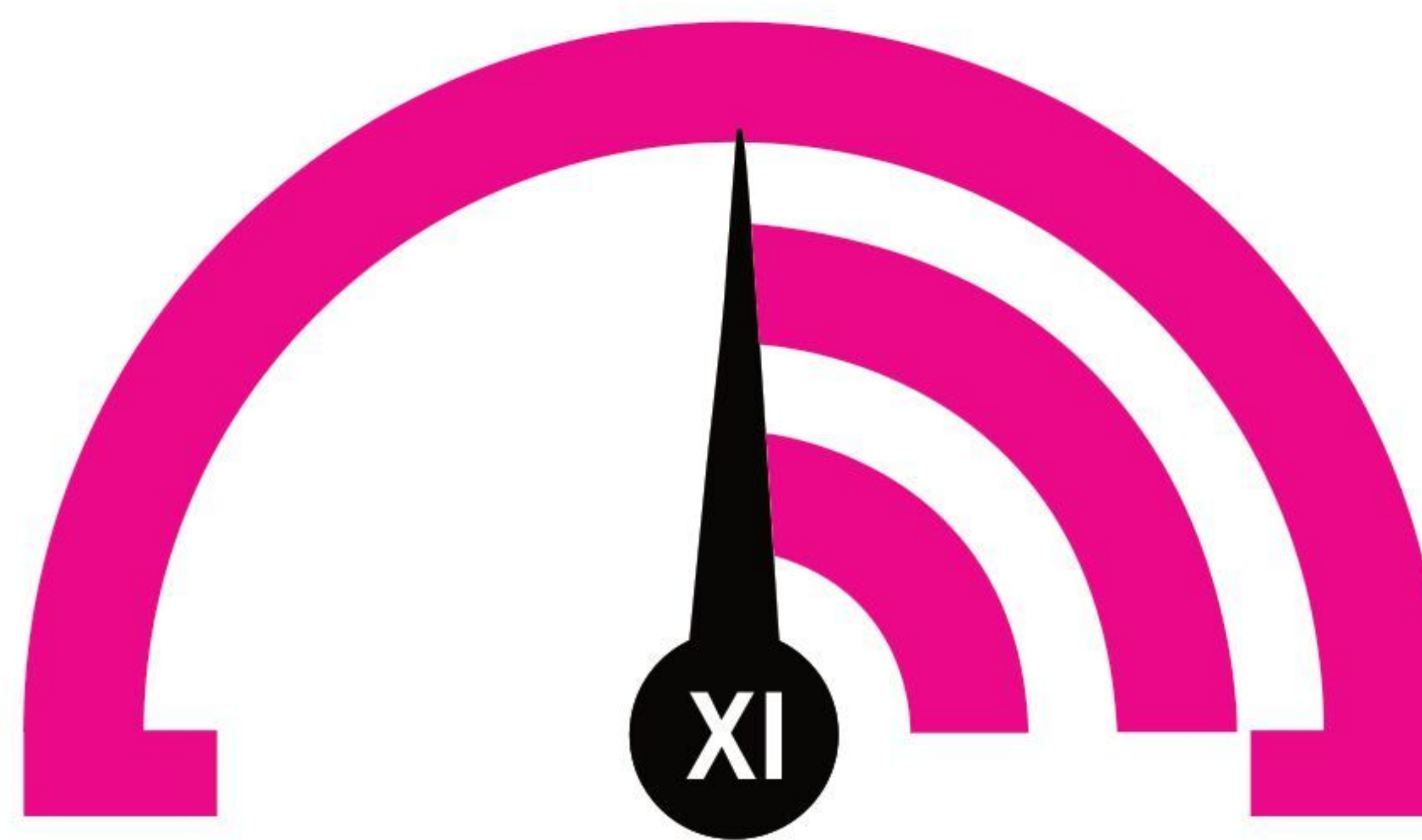
SAMURAI SUDOKU

ANSWER - NOVEMBER 2022



5	8	2	6	4	3	7	1	9				2	7	4	9	6	3	8	1	5
4	9	6	8	7	1	2	3	5				3	8	6	1	2	5	4	7	9
3	7	1	5	2	9	4	6	8				5	1	9	4	8	7	3	2	6
2	4	5	1	3	6	8	9	7				6	2	3	5	1	4	9	8	7
9	6	8	2	5	7	3	4	1				7	4	8	3	9	2	5	6	1
1	3	7	9	8	4	5	2	6				9	5	1	8	7	6	2	3	4
7	1	3	4	9	8	6	5	2	3	4	8	1	9	7	2	5	8	6	4	3
8	5	9	3	6	2	1	7	4	6	9	2	8	3	5	6	4	1	7	9	2
6	2	4	7	1	5	9	8	3	5	1	7	4	6	2	7	3	9	1	5	8
						3	4	7	8	2	5	9	1	6						
						5	6	9	1	3	4	2	7	8						
						8	2	1	9	7	6	3	5	4						
6	4	3	1	7	5	2	9	8	7	6	3	5	4	1	8	3	2	6	9	7
2	9	8	6	3	4	7	1	5	4	8	9	6	2	3	7	9	5	4	8	1
5	1	7	9	8	2	4	3	6	2	5	1	7	8	9	4	6	1	3	2	5
1	7	5	3	2	9	6	8	4				3	5	6	1	7	8	2	4	9
4	8	9	7	6	1	3	5	2				8	9	4	2	5	3	7	1	6
3	2	6	5	4	8	9	7	1				1	7	2	6	4	9	5	3	8
8	5	4	2	9	7	1	6	3				4	1	8	5	2	6	9	7	3
9	6	1	4	5	3	8	2	7				9	6	7	3	1	4	8	5	2
7	3	2	8	1	6	5	4	9				2	3	5	9	8	7	1	6	4

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Binomial Theorem

Time Taken : 60 Min.

Only One Option Correct Type

- If $C_r = \binom{n}{r}$, then $\sum_{r=1}^n r^2 \cdot C_r =$
 - $n(n-1)2^{n-1}$
 - $n(n+1)2^{n-1}$
 - $n(n+1)2^{n-2}$
 - $n(n+1)2^n$
- If $a = 99^{50} + 100^{50}$ and $b = 101^{50}$, then
 - $a = b$
 - $a > b$
 - $a < b$
 - none of these
- If $(1+x)^n = \sum_{r=0}^n C_r x^r$, then the sum of the products of the C_r taken two at a time represented by $\sum_{0 \leq i < j \leq n} C_i C_j$ is
 - $2^{2n-1} - \binom{2n-1}{n}$
 - $2^{2n-1} - \binom{2n}{n}$
 - $2^{2n} - \binom{2n}{n}$
 - $2^{2n} - \binom{2n}{n-1}$
- If $\frac{1}{(1-ax)(1-bx)} = a_0 + a_1x + a_2x^2 + \dots$, then $a_n =$
 - $\frac{a^{n+1} - b^{n+1}}{b-a}$
 - $\frac{b^{n+1} - a^{n+1}}{b-a}$
 - $\frac{b^n - a^n}{b-a}$
 - $\frac{a^n - b^n}{b-a}$
- $\sum_{r=0}^n (-1)^r \binom{n}{r} \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \dots \text{ to } m \text{ terms} \right]$ is equal to
 - $\frac{1-2^{-2mn}}{1-2^{-n}}$
 - $\frac{1+2^{-mn}}{1-2^{-n}}$
 - $\frac{1-2^{-mn}}{2^n-1}$
 - $\frac{1-2^{-mn}}{1+2^n}$

- For any positive integers with $n \geq m$,

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots + \binom{m}{m} =$$
 - $\binom{n+1}{m}$
 - $\binom{n+1}{m+1}$
 - $\binom{n}{m+1}$
 - $\binom{n+m}{m}$

One or More Than One Option(s) Correct Type

- If the coefficient of x^8 in the expansion of $\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!}\right)^2$ is $\frac{1}{M}$, then a divisor of M is
 - 2
 - 3
 - 5
 - 7
- If the 9th term in the expansion of $\left(\frac{\sqrt{10}}{(\sqrt{x})^{5 \log_{10} x}} + x \cdot x^{\frac{1}{2 \log_{10} x}}\right)^{10}$ is 450, then $x =$
 - 10
 - 100
 - $10^{-1/5}$
 - $10^{-2/5}$

UNSCRAMBLED WORDS

NOVEMBER 2022

- | | |
|------------------|-----------------|
| 1-c- INTEGRATION | 2-d- ELLIPSE |
| 3-f- EQUATION | 4-a- SURJECTIVE |
| 5-b- ADJOINTS | 6-e- TANGENT |
| 7-j- INCIRCLE | 8-g- EXCENTRE |
| 9-i- ASYMPTOTE | 10-h- DIAMETER |

Winner : Raayan Halder

9. The coefficient of x^4 in the expansion of $(1 + x + x^2)^6$ is divisible by

(a) 2 (b) 3
(c) 5 (d) 7

10. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$, then

(a) $a_1 = 20$ (b) $a_2 = 210$
(c) $a_4 = 8085$ (d) $a_{20} = 2^2 \times 3^7 \times 7$.

11. If the coefficients of x^{-2} and x^{-4} in the expansion

of $\left(x^3 + \frac{1}{2x^3}\right)^{18}$, $(x > 0)$, are m and n respectively,

then $\frac{m}{n}$ is equal to

(a) 27 (b) 182 (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

12. If $S = C_0^2 + \frac{C_1^2}{C_0} + 2 \cdot \frac{C_2^2}{C_1} + 3 \cdot \frac{C_3^2}{C_2} + \dots + n \cdot \frac{C_n^2}{C_{n-1}}$ where $C_r = {}^nC_r$, then

(a) $(n + 2)$ divides $S + n$
(b) 2^{n-1} divides $S + n$
(c) S is even if n is even
(d) S is even for all n

13. For the expansion of $\left(x^3 + 3 \cdot 2^{-\log \sqrt{2}} \sqrt{x^3}\right)^5$, which of the following is/are correct?

(a) Coefficient of x^2 is non zero
(b) Coefficient of x^2 is zero
(c) Coefficient of x^{-3} is non zero
(d) Ratio of coefficient of x^3 to that of x^{-3} is 1 : 3.

Comprehension Type

Paragraph for Q. No. 14 and 15

Let $\sum_{r=0}^{16} C_r x^r = (1 + x)^{16}$. Then

14. $C_0 + C_4 + C_8 + C_{12} + C_{16} =$

(a) $2^8 (2^8 - 1)$ (b) $2^8 (2^8 + 1)$
(c) $2^7 (2^7 - 1)$ (d) $2^7 (2^7 + 1)$

15. $C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15} =$

(a) $\frac{1}{3}(2^{16} + 1)$ (b) $\frac{1}{3}(2^{16} - 1)$
(c) $\frac{1}{3}(2^{15} + 1)$ (d) $\frac{1}{3}(2^{15} - 1)$

Matrix Match Type

16. Match the following columns:

Column-I		Column-II	
(P)	If ${}^{(n+1)}C_4 + {}^{(n+1)}C_3 + {}^{n+2}C_3 > {}^{(n+3)}C_3$, then possible value/values of n is/are	(1)	4
(Q)	The remainder when $(3053)^{456} - (2417)^{333}$ is divided by 9 is less than	(2)	5
(R)	The digit in the unit place of the number $183! + 3^{183}$ is greater than	(3)	6
(S)	If sum of the coefficients of the first, second and third terms of the expansion of $\left(x^2 + \frac{1}{x}\right)^m$ is 46, then the index of the term that does not contain x is greater than	(4)	7

	P	Q	R	S
(a)	1,3,4	1,2	1,2,3	3,4
(b)	2,3,4	1,2,3	2,3,4	1,3
(c)	1,2	3,4	1,4	2,3
(d)	2,3,4	1,2,3,4	1,2,3	1,2

Numerical Answer Type

17. The number of terms which are free from radical signs in the expansion of $(y^{1/5} + x^{1/10})^{55}$ is _____.
18. If $(2r + 3)^{\text{th}}$ and $(r - 1)^{\text{th}}$ terms in the expansion of $(1 + x)^{15}$ have equal coefficients, then r is _____.
19. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n equals _____.
20. If the sum of the two middle coefficients of $(1 + x)^9$ is xyz then $x + y + z$ equals _____.



Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted
No. of questions correct
Marks scored in percentage

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

mtg

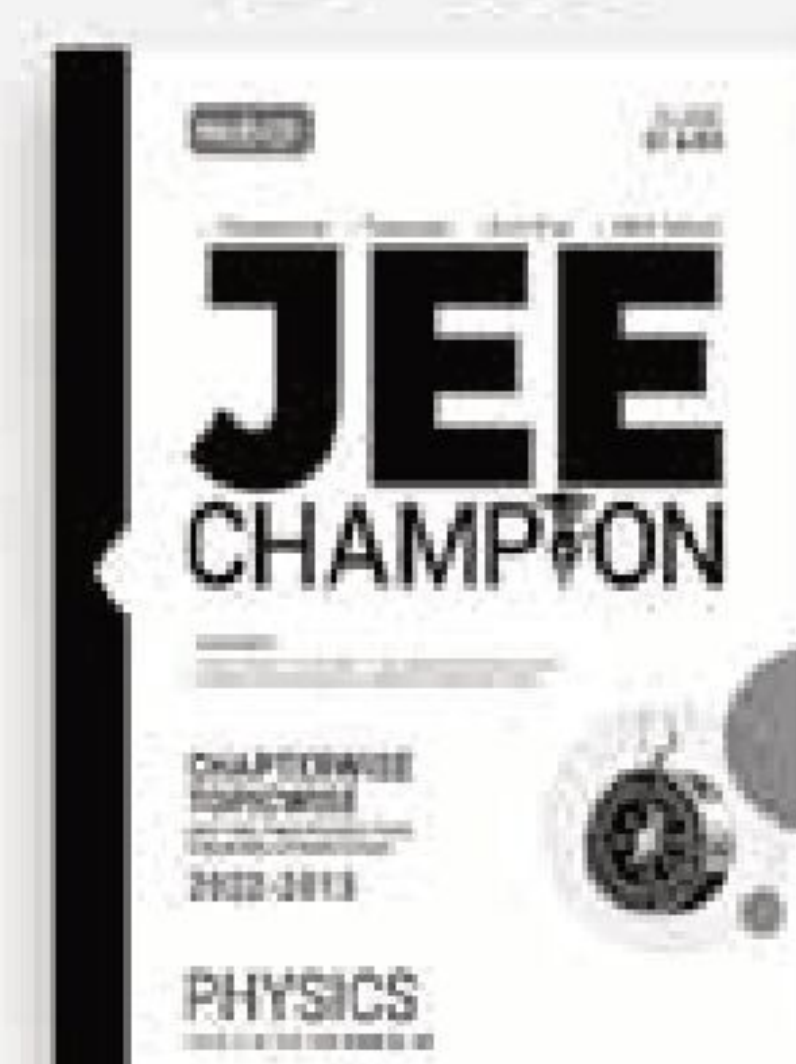
For Enquiries
0124-6601200

Find all your
QUESTIONS here,
before getting them
in your exam.

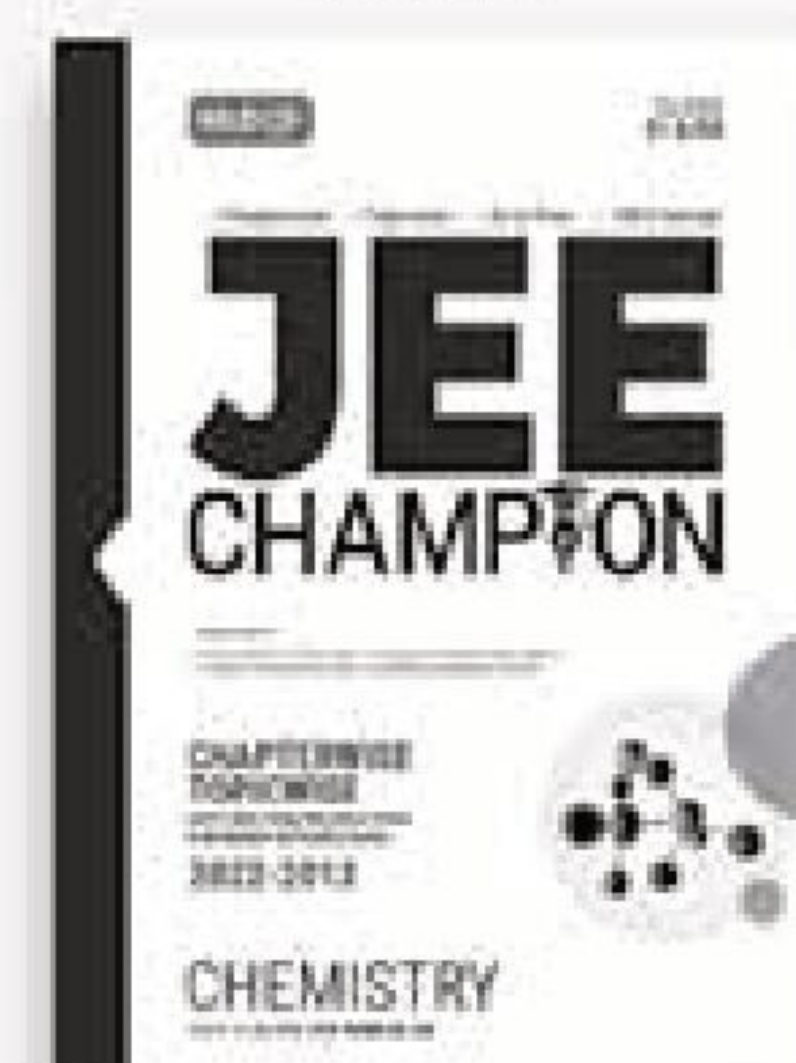
By not studying
MORE but
ACCURATE...



Our Books



₹ 900/-



₹ 900/-



₹ 900/-

JEE 2023 exams are nearing and all the students must be in the stage of preparation with full dedication and hard work. But have you put your knowledge to test? With our 41 years of experience, we would suggest you to put yourself through MTG's JEE Champion Series so that you can be sure about your level of preparation and find your core strengths and capabilities. The best part is you study at a pace you're comfortable with. Because it's all chapters, topicwise.



Scan to
explore & buy more
JEE Books on mtg.in



CBSE warm-up!

CLASS-XII

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2022-23.

Series-5

Differential Equations

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin \frac{dy}{dx} \text{ is}$$

- (a) 1 (b) 2
(c) 3 (d) Not defined

2. The degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2} \text{ is}$$

- (a) 4 (b) 3/2
(c) Not defined (d) 2

3. The order and degree respectively of the differential

$$\text{equation } \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0 \text{ are}$$

- (a) 2 and 4 (b) 2 and 2
(c) 2 and 3 (d) 3 and 3

4. Integrating factor of differential equation

$$\frac{dy}{dx} + y \tan x - \sec x = 0 \text{ is}$$

- (a) $\cos x$ (b) $\sec x$ (c) $e^{\cos x}$ (d) $e^{\sec x}$

5. The number of arbitrary constants in the particular solution of a differential equation of second order is
- (a) 0 (b) 1 (c) 2 (d) 3

6. Solution of differential equation $xdy - ydx = 0$ represents

- (a) a rectangular hyperbola
(b) parabola whose vertex is at origin
(c) straight line passing through origin
(d) a circle whose centre is at origin

7. The general solution of $e^x \cos y dx - e^x \sin y dy = 0$ is

- (a) $e^x \cos y = k$ (b) $e^x \sin y = k$
(c) $e^x = k \cos y$ (d) $e^x = k \sin y$

8. The general solution of $\frac{dy}{dx} = 2xe^{x^2-y}$ is

- (a) $e^{x^2-y} = C$ (b) $e^{-y} + e^{x^2} = C$
(c) $e^y = e^{x^2} + C$ (d) $e^{x^2+y} = C$

9. Which of the following is not a homogeneous function of x and y ?

- (a) $x^2 + 2xy$ (b) $2x + y$
(c) $\cos^2(y/x) + (y/x)$ (d) $\sin x - \cos y$

10. Which of the following is a homogeneous differential equation?

- (a) $(x^3 + x^2 + x) \frac{dy}{dx} = 2x^3 + x$
(b) $x^2 dy = (2xy + y^2) dx$
(c) $\frac{x}{1+x^2} dx = \frac{y}{1+y^2} dy$
(d) None of these

11. The integrating factor of the differential equation

$$y^2 \frac{dx}{dy} + x - \frac{1}{y} = 0 \text{ is}$$

- (a) $e^{\frac{1}{y}}$ (b) $e^{-\frac{1}{y}}$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

12. Integrating factor of the differential equation

$$(1-x^2) \frac{dy}{dx} - xy = 1 \text{ is}$$

- (a) $-x$ (b) $\frac{x}{1+x^2}$
(c) $\sqrt{1-x^2}$ (d) $\frac{1}{2} \log(1-x^2)$

13. If m and n are degree and order of $(1+y_1^2)^{2/3} = y_2$, then the value of $\frac{m+n}{m-n}$ is

- (a) 3 (b) 4 (c) 5 (d) 2

14. The order and degree respectively of the differential

$$\text{equation } x^2 \frac{d^2 y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^4 \text{ are}$$

- (a) 2, 1 (b) 1, 2 (c) 2, 3 (d) 4, 2

15. The order and degree respectively of the differential

$$\text{equation } \left(\frac{d^2 y}{dx^2} \right)^5 + 4 \frac{\left(\frac{d^2 y}{dx^2} \right)^3}{\frac{d^3 y}{dx^3}} + \frac{d^3 y}{dx^3} = \sin x, \text{ are}$$

- (a) 1, 3 (b) 2, 3 (c) 3, 1 (d) 3, 2

16. Integrating factor of $(2x - 10y^3) \frac{dy}{dx} + y = 0$ is

- (a) y^3 (b) y (c) $-y$ (d) y^2

17. The number of solution of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

- (a) none (b) one (c) two (d) infinite

18. If $\frac{dy}{dx} = \frac{y+x \tan \frac{y}{x}}{x}$ is homogeneous differential equation, then $\sin\left(\frac{y}{x}\right)$ is equal to

- (a) Cx^3 (b) Cx^2 (c) Cx (d) $\log x$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A) : ' x ' is not an integrating factor for the differential equation $x \frac{dy}{dx} + 2y = e^x$.

$$\text{Reason (R) : } x \left(x \frac{dy}{dx} + 2y \right) = \frac{d}{dx} (x^2 y).$$

20. Assertion (A) : The equation of curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$.

$$\text{Reason (R) : The solution of differential equation } \left(\frac{dy}{dx} \right)^2 - \left(\frac{dy}{dx} \right) (e^x + e^{-x}) + 1 = 0 \text{ is } y = c_1 e^x + c_2 e^{-x}.$$

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Find the general solution of the differential equation : $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

22. Find the general solution of the differential equation : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

OR

Solve the following differential equation:

$$\frac{dy}{dx} + y = \cos x - \sin x$$

23. Write the sum of order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$.

24. Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

OR

Solve the differential equation $(1 + y^2) \tan^{-1} x \, dx + 2y(1 + x^2)dy = 0$.

25. Find the solution of the differential equation $\cos x \sin y \, dx + \sin x \cos y \, dy = 0$.

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Solve the differential equation

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

27. Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xydy = 0$, given that $y = 0$, when $x = 1$.

OR

Find the general solution of the differential equation $\frac{dy}{dx} + yg'(x) = g(x)g'(x)$, where $g(x)$ is a function of x .

28. Find the particular solution of the differential equation $e^x \sqrt{1 - y^2} \, dx + \frac{y}{x} \, dy = 0$, given that $y = 1$, when $x = 0$.

OR

Find the equation of curve passing through the point $(1, 0)$ and whose slope is $\frac{y-1}{x^2+x}$.

29. Find the particular solution of the differential equation $x \frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$, given that $y(1) = 0$.
30. Find the general solution of the differential equation $x(y^3 + x^3) \, dy = (2y^4 + 5x^3y)dx$.

OR

Solve the following differential equation : $(y - \sin^2 x)dx + \tan x \, dy = 0$

31. Solve the following differential equation : $2xy \, dx + (x^2 + 2y^2)dy = 0$

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Find the particular solution of the differential equation $(x^2 + 1) \frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 0$.

OR

Find the general solution of the differential equation $(x - y) \frac{dy}{dx} = x + 2y$.

33. The slope of the tangent to a curve at any point (x, y) on it is given by $\frac{y}{x} - \cot \frac{y}{x} \cdot \cos \frac{y}{x}$, $(x > 0, y > 0)$ and the curve passes through the point $(1, \pi/4)$. Find the equation of the curve.

34. Find the particular solution of the differential equation $(\tan^{-1} y - x) \, dy = (1 + y^2)dx$, given that $x = 1$ when $y = 0$.

OR

Solve the following differential equation :

$$\left[y - x \cos\left(\frac{y}{x}\right) \right] dy + \left[y \cos\left(\frac{y}{x}\right) - 2x \sin\left(\frac{y}{x}\right) \right] dx = 0$$

35. Solve the following differential equation : $\left(\sqrt{1 + x^2 + y^2 + x^2 y^2} \right) dx + xy \, dy = 0$

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. **Case Study 1:** If an equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x , then such equation is known as linear differential equation. Its solution is given by $y \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.}) \, dx + c$, where $\text{I.F.} = e^{\int P dx}$. Now, suppose the given equation is $(1 + \sin x) \frac{dy}{dx} + y \cos x + x = 0$.

Based on the above information, answer the following questions.

- (i) Find the value of integrating factor.

(ii) Find the general solution of given differential equation.

(iii) Find y if $y(0) = 1$

OR

Find $y\left(\frac{\pi}{2}\right)$.

37. Case Study 2 : In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective measured the body temperature and found it to be 70°F . Two hours later, the detective measured the body temperature again and found it to be 60°F , where the room temperature is 50°F . Also, it is given the body temperature at the time of death was normal, i.e., 98.6°F .



Let T be the temperature of the body at any time t and initial time is taken to be 8 p.m.

Based on the above information, answer the following questions.

- By Newton's law of cooling, $\frac{dT}{dt} = kf(t)$, where k is a constant, then find $f(T)$.
- Find the value of T at any time t .
- If it is given that $\log_e(2.43) = 0.88789$ and $\log_e(0.5) = -0.69315$, then find the time at which the murder occur.

OR

Find the value of $\lim_{x \rightarrow \infty} T$.

38. Case Study 3 : If the equation is of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ or $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in

x and y , then put $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$, so that

the dependent variable y is changed to another variable v and then apply variable separable method. Based on the above information, answer the following questions.

- Find the general solution of $x^2 \frac{dy}{dx} = x^2 + xy + y^2$.
- What is the solution of the differential equation $2xy \frac{dy}{dx} = x^2 + 3y^2$?

SOLUTIONS

1. (d) : Since, the given differential equation is not a polynomial in $\frac{dy}{dx}$. Therefore, its degree is not defined.

2. (d) : We have, $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

On squaring both sides, we get

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

The highest order derivative is $\frac{d^2y}{dx^2}$ and the highest power raised to $\frac{d^2y}{dx^2}$ is 2.

So, the degree of the given differential equation is 2.

3. (a) : We have, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$

$$\Rightarrow \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get $\left(\frac{dy}{dx}\right)^{1/2} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^2$

On squaring both sides again, we get

$$\frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$$

$\therefore$ Order = 2, degree = 4

4. (b) : We have, $\frac{dy}{dx} + y \tan x - \sec x = 0$

$$\text{or } \frac{dy}{dx} + y \tan x = \sec x$$

This is linear differential equation of the type

$$\frac{dy}{dx} + Py = Q, \text{ with } P = \tan x, Q = \sec x$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \tan x dx} = e^{(\log \sec x)} = \sec x$$

5. (a) : In the particular solution of a differential equation of any order, there is no arbitrary constant because in the particular solution of any differential equation, we remove all the arbitrary constant by substituting some particular values.



Recipe for Success

“Do one thing every day that scares you.”

– Eleanor Roosevelt

6. (c) : We have, $xdy - ydx = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$

Integrating both sides, we get

$$\log y = \log x + \log C \Rightarrow y = Cx,$$

which is equation of a straight line passing through origin.

7. (a) : We have, $e^x \cos y dx - e^x \sin y dy = 0$

$$\Rightarrow e^x \cos y dx = e^x \sin y dy \Rightarrow dx = \tan y dy$$

Integrating both sides, we get

$$x = \ln \sec y + \ln k$$

$$\Rightarrow x = \ln(k \sec y) \Rightarrow e^x = k \sec y \Rightarrow e^x \cos y = k$$

8. (c) : We have, $\frac{dy}{dx} = 2x e^{x^2 - y}$

$$\Rightarrow \frac{dy}{dx} = 2x e^{x^2} \cdot e^{-y} \Rightarrow e^y dy = 2x e^{x^2} dx$$

Integrating both sides, we get $e^y = e^{x^2} + C$

9. (d) : Since, $\sin x$ and $\cos y$ can't be expressed in the form $x^n g(y/x)$ or $y^n h(x/y)$.

$\therefore$ It is not a homogeneous function.

10. (b) : We have,

$$\frac{dy}{dx} = \frac{2xy + y^2}{x^2} \text{ or } f(x, y) = \frac{2xy + y^2}{x^2}$$

$$\begin{aligned} \text{Now, } f(\lambda x, \lambda y) &= \frac{2(\lambda x)(\lambda y) + \lambda^2 y^2}{(\lambda x)^2} \\ &= \frac{\lambda^2 (2xy + y^2)}{\lambda^2 x^2} = \lambda^0 \frac{(2xy + y^2)}{x^2} = \lambda^0 f(x, y) \end{aligned}$$

$\therefore$ It is a homogeneous differential equation.

11. (b) : We have, $y^2 \frac{dx}{dy} + x - \frac{1}{y} = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}, \text{ which is a linear differential}$$

equation of the form $\frac{dx}{dy} + Px = Q$, where

$$P = \frac{1}{y^2}, Q = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

12. (c) : We have, $(1 - x^2) \frac{dy}{dx} - xy = 1$

$$\Rightarrow \frac{dy}{dx} - \frac{x}{1 - x^2} y = \frac{1}{1 - x^2}$$

This is a linear differential equation of the type

$$\frac{dy}{dx} + Py = Q \text{ where } P = \frac{-x}{1 - x^2}, Q = \frac{1}{1 - x^2}$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{-\int \frac{x}{1 - x^2} dx} = e^{\frac{1}{2} \int \frac{-2x}{1 - x^2} dx}$$

$$= e^{\frac{1}{2} \log(1 - x^2)} = e^{\log \sqrt{1 - x^2}} = \sqrt{1 - x^2}$$

13. (c) : Given, $(1 + y_1^2)^{2/3} = y_2 \Rightarrow (1 + y_1^2)^2 = y_2^3$
So, order (n) = 2 and degree (m) = 3.

$$\text{Thus, } \frac{m+n}{m-n} = \frac{5}{1} = 5$$

14. (a) : The given differential equation is

$$x^2 \frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^4$$

$\therefore$ Its order is 2 and degree is 1.

15. (d) : The given equation can be written as

$$\left(\frac{d^2 y}{dx^2} \right)^5 \frac{d^3 y}{dx^3} + 4 \left(\frac{d^2 y}{dx^2} \right)^3 + \left(\frac{d^3 y}{dx^3} \right)^2 = \sin x \left(\frac{d^3 y}{dx^3} \right)$$

$\therefore$ Order = 3, degree = 2

16. (d) : Given differential equation is

$$(2x - 10y^3) \frac{dy}{dx} + y = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x - 10y^3} \Rightarrow \frac{dx}{dy} = \frac{2x - 10y^3}{-y} = \frac{-2x}{y} + 10y^2$$

$$\Rightarrow \frac{dx}{dy} + \frac{2}{y} x = 10y^2$$

$$\text{I.F.} = e^{\int \frac{2}{y} dy} = e^{2 \log y} = e^{\log y^2} = y^2$$

17. (b) : We have, $\frac{dy}{dx} = \frac{y+1}{x-1} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x-1}$

Integrating both sides, we get

$$\log(y+1) = \log(x-1) - \log C \Rightarrow C(y+1) = (x-1)$$

$$\text{Given, } y(1) = 2 \text{ i.e., } x = 1; y = 2 \therefore 3C = 0 \Rightarrow C = 0$$

$\therefore$ Required solution is $x - 1 = 0 \Rightarrow x = 1$

Hence, only one solution exist.

18. (c) : We have, $\frac{dy}{dx} = \frac{y + x \tan y/x}{x} = \frac{y}{x} + \tan \frac{y}{x} \dots (i)$,

which is clearly a homogeneous differential equation

$$\text{Putting } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore \text{From (i), we have, } v + x \frac{dv}{dx} = v + \tan v \Rightarrow \frac{1}{\tan v} dv = \frac{dx}{x}$$

$$\Rightarrow \log \sin v = \log x + \log C \Rightarrow \sin v = xC \Rightarrow \sin \left(\frac{y}{x} \right) = xC.$$

19. (b): $\frac{dy}{dx} + \frac{2}{x}y = \frac{e^x}{x}$

I.F. = $e^{\int \frac{2}{x} dx} = e^{\log x^2} = x^2$

⇒ Statement-I is true.

Now, $\frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + y \cdot 2x = x \left(x \frac{dy}{dx} + 2y \right)$

⇒ Statement-II is true.

20. (b): Statement-I

$$\int dy = \int \left(x + \frac{1}{x^2} \right) dx \Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + C$$

It passes through (3, 9).

$$\therefore 9 = \frac{9}{2} - \frac{1}{3} + C \Rightarrow C = 9 - \frac{9}{2} + \frac{1}{3} = \frac{29}{6}$$

$$\Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$$

$$\therefore 6xy = 3x^3 + 29x - 6$$

⇒ Statement-I is true.

Statement-II

Differential equation can be written as

$$\left(\frac{dy}{dx} - e^x \right) \left(\frac{dy}{dx} - e^{-x} \right) = 0$$

$$\Rightarrow \frac{dy}{dx} = e^x \text{ or } \frac{dy}{dx} = e^{-x}$$

$$\Rightarrow \int dy = \int e^x dx \text{ or } \int dy = \int e^{-x} dx$$

$$\Rightarrow y = e^x + c_1 \text{ or } y = -e^{-x} + c_2$$

$$\Rightarrow y = c_1 e^x + c_2 e^{-x} \text{ will satisfy the above equation.}$$

⇒ Statement-II is true.

21. We have, $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy \Rightarrow \frac{d(\tan x)}{\tan x} = -\frac{d(\tan y)}{\tan y}$$

Integrating on both sides we get

$$\Rightarrow \log(\tan x) = -\log(\tan y) + \log C \Rightarrow \tan x \tan y = C$$

22. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$\Rightarrow dy = \frac{(e^x + x^2)}{e^y} dx \Rightarrow e^y dy = (e^x + x^2) dx$$

Integrating on both sides, we get

$$\Rightarrow \int e^y dy = \int (e^x + x^2) dx$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + C, \text{ which is required solution.}$$

OR

We have, $\frac{dy}{dx} + y = \cos x - \sin x$

This is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P=1, Q = \cos x - \sin x$$

$$\therefore \text{I.F.} = e^{\int dx} = e^x$$

The solution of given differential equation is

$$ye^x = \int e^x (\cos x - \sin x) dx + C$$

$$\Rightarrow ye^x = e^x \cos x + C \Rightarrow y = \cos x + Ce^{-x}$$

23. We have, $\frac{d}{dx} \left(\frac{dy}{dx} \right) = 5 \Rightarrow \frac{d^2 y}{dx^2} = 5$

$$\therefore \text{Order} = 2$$

$$\text{Degree} = 1$$

$$\therefore \text{Required sum} = 3$$

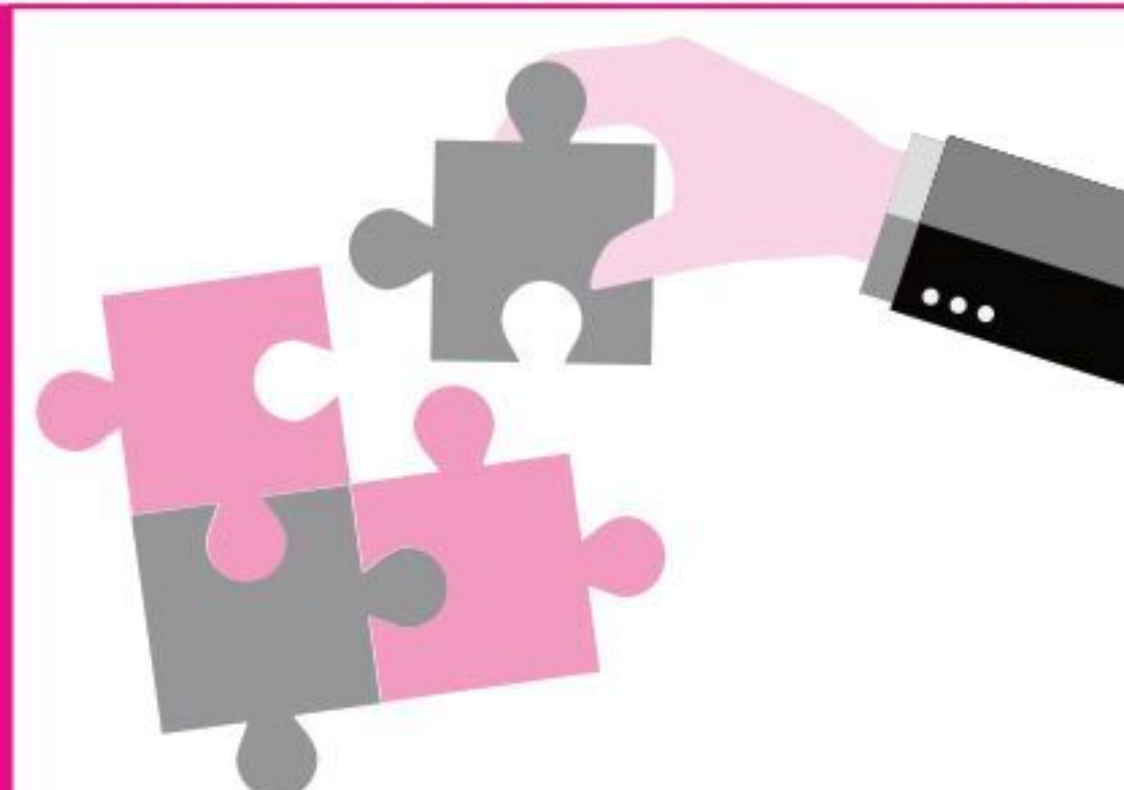
24. We have, $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

Integrating both sides, we get

$$\Rightarrow \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$$\Rightarrow -2^{-y} + 2^{-x} = C \log 2 = k(\text{say}) \Rightarrow 2^{-x} - 2^{-y} = k$$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

3	4-		2÷	10+	
3÷	10+			5×	
		2-			
3-	24×	24×	3+		2-
			3-	2÷	
4-					6

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

OR

We have, $(1 + y^2)\tan^{-1}x dx + 2y(1 + x^2)dy = 0$

$$\Rightarrow \frac{\tan^{-1}x dx}{1 + x^2} = -\frac{2y}{1 + y^2} dy$$

Integrating both sides, we get

$$\begin{aligned} \frac{1}{2}(\tan^{-1}x)^2 &= -\log(1 + y^2) + C \\ \Rightarrow \frac{1}{2}(\tan^{-1}x)^2 + \log(1 + y^2) &= C \end{aligned}$$

25. We have, $\cos x \sin y dx + \sin x \cos y dy = 0$

$$\Rightarrow \cos x \sin y dx = -\sin x \cos y dy$$

$$\Rightarrow \cot x dx = -\cot y dy$$

Integrating both sides, we get

$$\log \sin x = -\log \sin y + \log C$$

$$\Rightarrow \log \sin x + \log \sin y = \log C \Rightarrow \sin x \cdot \sin y = C$$

26. We have, $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2}$$

It is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{x \log x}, Q = \frac{2}{x^2}$$

$$\therefore \text{I.F.} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$

$\therefore$ Solution of the differential equation is given by

$$\begin{aligned} y(\log x) &= \int (\log x) \frac{2}{x^2} dx + C \\ \Rightarrow y(\log x) &= \log x \int \frac{2}{x^2} dx - \int \left(\frac{d}{dx}(\log x) \int \frac{2}{x^2} dx \right) dx + C \\ \Rightarrow y(\log x) &= \log x \left(-\frac{2}{x} \right) - \int \frac{1}{x} \left(-\frac{2}{x} \right) dx + C \\ \Rightarrow y(\log x) &= \log x \left(-\frac{2}{x} \right) + \int \frac{2}{x^2} dx + C \\ \Rightarrow y(\log x) &= \log x \left(-\frac{2}{x} \right) - \frac{2}{x} + C \end{aligned}$$

27. We have, $(1 - y^2)(1 + \log x) dx + 2xy dy = 0$

$$\Rightarrow (1 - y^2)(1 + \log x) dx = -2xy dy$$

$$\Rightarrow \frac{(1 + \log x)}{x} dx = -\frac{2y}{1 - y^2} dy$$

On integrating both sides, we get

$$\int \frac{1 + \log x}{x} dx = \int -\frac{2y}{1 - y^2} dy$$

$$\frac{(1 + \log x)^2}{2} = \log |1 - y^2| + C$$

When $x = 1, y = 0$

$$\therefore \frac{(1 + \log 1)^2}{2} = \log(1) + C \Rightarrow C = \frac{1}{2}$$

$$\Rightarrow \frac{(1 + \log x)^2}{2} = \log |1 - y^2| + \frac{1}{2}$$

$\Rightarrow (1 + \log x)^2 = 2 \log |1 - y^2| + 1$ is the required solution.

OR

We have $\frac{dy}{dx} + yg'(x) = g(x)g'(x)$

Clearly, it is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = g'(x) \text{ and } Q = g(x)g'(x).$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int g'(x) dx} = e^{g(x)}$$

$$\text{The solution is } y \cdot e^{g(x)} = \int e^{g(x)} g(x) g'(x) dx + C' \quad \dots(i)$$

$$\text{Let } I = \int e^{g(x)} g(x) g'(x) dx$$

$$\text{Putting } g(x) = t \Rightarrow g'(x) dx = dt$$

$$\begin{aligned} \therefore I &= \int te^t dt = t \int e^t dt - \int \left[\frac{d}{dt}(t) \int e^t dt \right] dt = te^t - \int e^t dt \\ &= te^t - e^t = g(x)e^{g(x)} - e^{g(x)} \end{aligned}$$

$$\therefore \text{From (i), we have } y \cdot e^{g(x)} = g(x)e^{g(x)} - e^{g(x)} + C'$$

$$\Rightarrow y \cdot e^{g(x)} + e^{g(x)} - g(x)e^{g(x)} = C'$$

$$\Rightarrow y + 1 - g(x) = C'e^{-g(x)}$$

$$\Rightarrow \log[y + 1 - g(x)] = -g(x) + \log C'$$

$$\Rightarrow g(x) + \log[y + 1 - g(x)] = C, \text{ where } C = \log C'$$

28. We have, $e^x \sqrt{1 - y^2} dx + \frac{y}{x} dy = 0$

$$x e^x dx + \frac{y}{\sqrt{1 - y^2}} dy = 0$$

Integrating both sides, we get

$$x \cdot e^x - \int 1 \cdot e^x dx - \frac{1}{2} \int (1 - y^2)^{-\frac{1}{2}} (-2y) dy = C$$

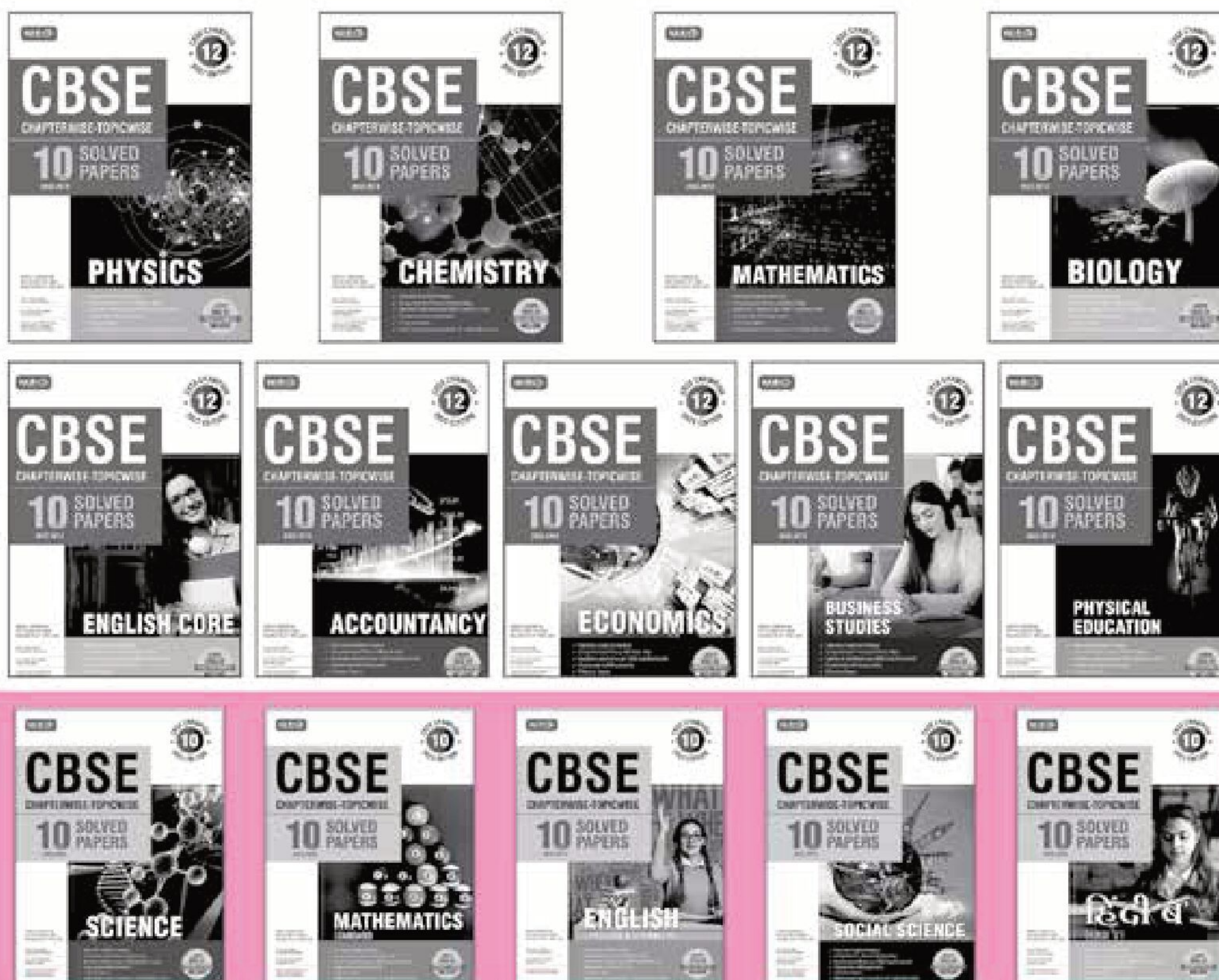
$$\Rightarrow x e^x - e^x - \frac{1}{2} \frac{(1 - y^2)^{\frac{1}{2}}}{1/2} = C$$

$$\Rightarrow e^x (x - 1) - \sqrt{1 - y^2} = C$$

When $x = 0, y = 1,$

$$\therefore e^0 (0 - 1) - \sqrt{1 - 1} = C \Rightarrow C = -1$$

CBSE CHAPTERWISE TOPICWISE 10 SOLVED PAPERS (2022-2013)



CBSE CHAMPION Chapterwise-Topicwise Solved Papers Series contains topicwise questions and solutions asked over last decade in CBSE-Board examination.

Examination papers for Class 10 and 12 Boards are based on a certain pattern. To excel, studying right is therefore more important than studying hard, which is why we created this series.

Questions are supported with topicwise graphical analysis of previous years CBSE Board questions as well as comprehensive and lucid theory. Questions from Term I, Term II, Delhi, All India and Foreign papers are included.

Important feature of these books is that the solutions to all the questions have been given according to CBSE marking scheme with Preparatory Tools and Topper's Answers. Practice papers are also supplemented.

HIGHLIGHTS OF 2022-23 EDITION

- Based on Reduced Syllabus
- Topicwise Graphical Analysis
- Quick Recap with Brain Map
- Previous 10 years (2022-2013) questions of CBSE
- CBSE Cognitive Level Tagging
- Key Points, Answer Tips, Concept Applied, Shortcuts, Alternating Methods, Commonly Made Mistakes provided
- CBSE Topper's Answers
- Past 3 years CBSE Sample Question Papers
- Chapterwise Self-Assessment
- 5 Practice Papers
- CBSE Sample Paper (Issued on 16th September 2022)



Available at all leading book shops throughout India.
For more information or for help in placing your order:
Call 0124-6601200 or email info@mtg.in
Visit www.mtg.in for latest offers and to buy online!

$$\therefore e^x(x-1) - \sqrt{1-y^2} = -1$$

$$\Rightarrow e^x(x-1) - \sqrt{1-y^2} + 1 = 0 \text{ is the required solution.}$$

OR

We have, $\frac{dy}{dx} = \frac{y-1}{x^2+x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$

$$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x(x+1)} \Rightarrow \frac{dy}{y-1} = \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$$

Integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$$

$$\Rightarrow \log(y-1) = \log x - \log(x+1) + \log C$$

$$\Rightarrow \log(y-1) = \log \left[\left(\frac{x}{x+1} \right) \cdot C \right]$$

$$\Rightarrow (y-1)(x+1) = xC$$

Since it passes through (1, 0), i.e., $x=1, y=0$

$$\therefore (-1)(1+1) = C \Rightarrow C = -2$$

$$\therefore \text{Required curve is } (y-1)(x+1) = -2x$$

$$\Rightarrow (y-1)(x+1) + 2x = 0$$

29. We have, $x \frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$... (i)

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{-1}{(1+x^2)x}$$

Clearly, it is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{x} \text{ and } Q = \frac{-1}{x(1+x^2)}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\log x} = x.$$

$\therefore$ Solution of (i) is

$$y \cdot x = \int \frac{-1}{x(1+x^2)} \cdot x dx + C$$

$$\Rightarrow yx = \int \frac{-1}{1+x^2} dx + C \Rightarrow yx = -\tan^{-1} x + C$$

Also, given $y(1) = 0$

$$\therefore 0 \times 1 = -\tan^{-1} 1 + C \Rightarrow C = \tan^{-1} 1 = \frac{\pi}{4}$$

$\therefore$ Particular solution of given differential equation is

$$yx = -\tan^{-1} x + \frac{\pi}{4}$$

30. We have, $x(y^3 + x^3) dy = (2y^4 + 5x^3y) dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2y^4 + 5x^3y}{xy^3 + x^4} \Rightarrow \frac{dy}{dx} = \frac{2(y/x)^4 + 5(y/x)}{(y/x)^3 + 1}$$

Put $v = \frac{y}{x} \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\therefore v + x \frac{dv}{dx} = \frac{2v^4 + 5v}{v^3 + 1}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v^4 + 5v}{v^3 + 1} - v = \frac{v^4 + 4v}{v^3 + 1}$$

$$\Rightarrow \frac{v^3 + 1}{v^4 + 4v} dv = \frac{dx}{x} \Rightarrow \int \frac{v^3 + 1}{v^4 + 4v} dv = \int \frac{dx}{x}$$

Putting $v^4 + 4v = t \Rightarrow (4v^3 + 4) dv = dt$

$$\therefore \frac{1}{4} \int \frac{dt}{t} = \int \frac{dx}{x} \Rightarrow \frac{1}{4} \log(v^4 + 4v) = \log x + \log C$$

$$\Rightarrow \frac{y^4 + 4yx^3}{x^8} = K \text{ where } C^4 = K$$

OR

$$(y - \sin^2 x) dx + \tan x dy = 0$$

$$(y - \sin^2 x) dx = -\tan x dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - \sin^2 x}{-\tan x} \Rightarrow \frac{dy}{dx} = \frac{\sin^2 x}{\tan x} - \frac{y}{\tan x}$$

$$\Rightarrow \frac{dy}{dx} = \sin x \cos x - y \cot x$$

$$\Rightarrow \frac{dy}{dx} + y \cot x = \sin x \cos x$$

So, it is a linear differential equation,

$$P = \cot x, Q = \sin x \cos x$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \cot x dx} = e^{\log |\sin x|} = \sin x$$

$$\text{General solution. : } y (\text{I.F.}) = \int Q (\text{I.F.}) dx + C$$

$$\Rightarrow y \cdot \sin x = \int \cos x \sin x \cdot \sin x dx + C$$

$$\Rightarrow y \cdot \sin x = \int \cos x \cdot \sin^2 x dx + C$$

$$\text{Let } \sin x = t \Rightarrow dx = \frac{dt}{\cos x}$$

$$\Rightarrow y \cdot \sin x = \int t^2 dt = \frac{t^3}{3} + C$$

$$\Rightarrow y \sin x = \frac{\sin^3 x}{3} + C \Rightarrow y = \frac{\sin^2 x}{3} + \frac{C}{\sin x}$$

31. The given differential equation is

$$2xy dx + (x^2 + 2y^2) dy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2 + 2y^2} = -\frac{2y/x}{1 + 2(y/x)^2} \quad \dots (1)$$

This is a homogeneous differential equation.

Put, $y = vx \Rightarrow \frac{dy}{dx} = v + \frac{dv}{dx} \cdot x$

Now, equation (1) becomes $v + x \frac{dv}{dx} = \frac{-2v}{1 + 2v^2}$

$$\Rightarrow x \frac{dv}{dx} = -v - \frac{2v}{1+2v^2} = \frac{-(3v+2v^3)}{1+2v^2}$$

$$\Rightarrow \frac{1+2v^2}{3v+2v^3} dv + \frac{dx}{x} = 0$$

On integrating, we get $\int \frac{1+2v^2}{3v+2v^3} dv + \int \frac{dx}{x} = \log c_1$

Putting $3v+2v^3 = t$

$$\Rightarrow (3+6v^2)dv = dt \Rightarrow (1+2v^2)dv = \frac{dt}{3}$$

$$\Rightarrow \frac{1}{3} \int \frac{dt}{t} + \log|x| = \log c_1$$

$$\Rightarrow \frac{1}{3} \log|t| + \log|x| = \log c_1$$

$$\Rightarrow |t^{\frac{1}{3}} \cdot x| = c_1 \Rightarrow t^{\frac{1}{3}} x = \pm c_1$$

$$\Rightarrow tx^3 = c, \text{ where } c = \pm c_1^3 \Rightarrow (3v+2v^3)x^3 = c$$

$$\Rightarrow 3vx^3 + 2v^3x^3 = c \Rightarrow 3x^2y + 2y^3 = c$$

This is the required solution of the given differential equation.

32. We have, $(x^2+1)\frac{dy}{dx} - 2xy = (x^4+2x^2+1)\cos x$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2+1}y = (x^2+1)\cos x, \text{ which is a linear}$$

differential equation of the form $\frac{dy}{dx} + Py = Q$, where

$$P = \frac{-2x}{x^2+1}, Q = (x^2+1)\cos x$$

$$\therefore \text{I.F.} = e^{\int \frac{-2x}{x^2+1} dx} = e^{-\log(x^2+1)} = (x^2+1)^{-1} = \frac{1}{x^2+1}$$

The solution is given by

$$y \cdot \frac{1}{x^2+1} = \int (x^2+1)\cos x \cdot \frac{1}{x^2+1} dx + C$$

$$\Rightarrow \frac{y}{x^2+1} = \int \cos x dx + C$$

$$\Rightarrow \frac{y}{x^2+1} = \sin x + C \quad \dots(i)$$

At $y(0) = 0$, i.e. $x = 0$, $y = 0$, we have

$$\frac{0}{0+1} = \sin 0 + C \Rightarrow C = 0$$

$\therefore$ From (i), we have

$$\therefore \frac{y}{x^2+1} = \sin x \Rightarrow y = (x^2+1)\sin x,$$

which is the required particular solution.

OR

We have, $(x-y)\frac{dy}{dx} = x+2y$

$$\Rightarrow \frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} \quad \dots(i)$$

This is a homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\therefore$ Equation (i) becomes, $v + x \frac{dv}{dx} = \frac{1+2v}{1-v}$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \frac{1-v}{1+v+v^2} dv = \frac{dx}{x}$$

SAMURAI SUDOKU



Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9×9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3×3 box should contain one of each digit.

The puzzle has a unique answer.

	3	4							4	1		6	9				
			7	6		9	2					7		5	3		
	6				1												
9			2					3				2		9			1
8	5				9							3		5			
						5	6					4	1			6	
6	7					8	1		2			6			2		1
				3						6			3			6	7
			4				3	7						5		3	
									8								1
						3						5					
						9											
4						7	8	3				6	2	8			3
9		5	2	8						3		9					
						2										5	
5	8		7		4							5			2		6
				2			1	5				1		5	9		8
	9		3											4			
				5			9	7				2				9	1
	7					4							1	7		4	
			6			2							4	6		8	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Integrating both sides, we get

$$\begin{aligned} \int \frac{-\frac{1}{2}(2v+1) + \frac{3}{2}}{v^2 + v + 1} dv &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \int \frac{2v+1}{v^2 + v + 1} dv + \frac{3}{2} \int \frac{dv}{\left(v + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \log|v^2 + v + 1| + \frac{3}{2} \cdot \frac{1}{\sqrt{3}/2} \tan^{-1} \left[\frac{v + \frac{1}{2}}{\sqrt{3}/2} \right] &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \log \left| \frac{y^2}{x^2} + \frac{y}{x} + 1 \right| + \sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3} \cdot x} \right) &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \log|y^2 + xy + x^2| + \frac{1}{2} \log|x^2| + \sqrt{3} \tan^{-1} \left(\frac{x+2y}{x\sqrt{3}} \right) &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \log|y^2 + xy + x^2| + \log|x| + \sqrt{3} \tan^{-1} \left(\frac{x+2y}{\sqrt{3}x} \right) &= \log|x| + C \\ \Rightarrow -\frac{1}{2} \log|y^2 + xy + x^2| + \sqrt{3} \tan^{-1} \left(\frac{x+2y}{\sqrt{3} \cdot x} \right) &= C. \end{aligned}$$

33. Let $y = f(x)$ be the given curve. Then, the slope of the tangent at $P(x, y)$ is $\frac{dy}{dx}$. But, the slope of the tangent

at P is given as $\frac{y}{x} - \cot \frac{y}{x} \cos \frac{y}{x}$.

Therefore,

$$\frac{dy}{dx} = \frac{y}{x} - \cot \frac{y}{x} \cos \frac{y}{x}$$

$$\text{Putting } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = v - \cot v \cos v \Rightarrow x \frac{dv}{dx} = -\cot v \cos v$$

$$\Rightarrow \sec v \tan v dv = -\frac{dx}{x}$$

On integrating both sides, we get

$$\Rightarrow \sec v = -\log|x| + C$$

$$\Rightarrow \sec \left(\frac{y}{x} \right) = -\log|x| + C$$

Since, the curve passes through $(1, \pi/4)$. i.e., $x = 1$, $y = \pi/4$

Therefore, $\sec \pi/4 = -\log 1 + C \Rightarrow C = \sqrt{2}$

$\therefore \sec \left(\frac{y}{x} \right) = -\log|x| + \sqrt{2}$, which is the required equation of the curve.

34. We have, $(\tan^{-1}y - x)dy = (1 + y^2)dx$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1 + y^2} \Rightarrow \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{\tan^{-1}y}{1 + y^2}$$

This is a linear differential equation of the form

$$\frac{dx}{dy} + Px = Q, \text{ where } P = \frac{1}{1 + y^2} \text{ and } Q = \frac{\tan^{-1}y}{1 + y^2}$$

$$\text{I.F.} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1}y}$$

$\therefore$ Required solution is,

$$x \cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y} \cdot \tan^{-1}y}{1 + y^2} dy + C \quad \dots(i)$$

$$\text{Put } \tan^{-1}y = t \Rightarrow \left(\frac{1}{1 + y^2} \right) dy = dt$$

$\therefore$ (i) becomes, $x \cdot e^{\tan^{-1}y} = \int e^t \cdot t dt + C$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = t \cdot e^t - \int 1 \cdot e^t dt + C$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = t \cdot e^t - e^t + C$$

$$\Rightarrow x \times e^{\tan^{-1}y} = \tan^{-1}y \times e^{\tan^{-1}y} - e^{\tan^{-1}y} + C$$

$$\Rightarrow x = \tan^{-1}y - 1 + Ce^{-\tan^{-1}y}$$

We get the solution as

$$x = \tan^{-1}y - 1 + Ce^{-\tan^{-1}y} \quad \dots(ii)$$

Now, putting $x = 1$, $y = 0$ in (ii), we get

$$1 = \tan^{-1}0 - 1 + Ce^{-\tan^{-1}(0)} \Rightarrow C = 2$$

So, required particular solution is $x = \tan^{-1}y - 1 + 2e^{-\tan^{-1}y}$.

OR

We have,

$$\left[y - x \cos \left(\frac{y}{x} \right) \right] dy + \left[y \cos \left(\frac{y}{x} \right) - 2x \sin \left(\frac{y}{x} \right) \right] dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x \sin \left(\frac{y}{x} \right) - y \cos \left(\frac{y}{x} \right)}{y - x \cos \left(\frac{y}{x} \right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \sin \left(\frac{y}{x} \right) - \frac{y}{x} \cos \left(\frac{y}{x} \right)}{\frac{y}{x} - \cos \left(\frac{y}{x} \right)} \quad \dots(i)$$

This is a homogeneous differential equation.

$$\therefore \text{ Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$\therefore$ Equation (i) becomes,

$$v + x \frac{dv}{dx} = \frac{2 \sin v - v \cos v}{v - \cos v} \Rightarrow x \frac{dv}{dx} = \frac{2 \sin v - v \cos v}{v - \cos v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2 \sin v - v^2}{v - \cos v} \Rightarrow \frac{v - \cos v}{2 \sin v - v^2} dv = \frac{dx}{x}$$

$$\Rightarrow \frac{-1}{2} \frac{(2 \cos v - 2v)}{2 \sin v - v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{-1}{2} \ln(2 \sin v - v^2) = \ln x + C_1$$

$$\Rightarrow \ln x^2 + 2C_1 + \ln \left(2 \sin \frac{y}{x} - \frac{y^2}{x^2} \right) = 0$$

$$\Rightarrow \ln \left[x^2 \left(2 \sin \frac{y}{x} - \frac{y^2}{x^2} \right) \right] = -2C_1$$

$$\Rightarrow 2x^2 \sin \frac{y}{x} - y^2 = e^{-2C_1} = C \text{ (say),}$$

which is the required solution.

35. We have, $\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy \frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{(1 + x^2)(1 + y^2)}}{xy}$$

$$\Rightarrow \int \frac{y}{\sqrt{1 + y^2}} dy = - \int \frac{\sqrt{1 + x^2}}{x^2} x dx$$

$$\text{Putting } 1 + x^2 = v^2 \Rightarrow 2x dx = 2v dv$$

$$\Rightarrow \frac{1}{2} \int \frac{2y}{\sqrt{1 + y^2}} dy = - \int \frac{v^2}{v^2 - 1} dv$$

$$\Rightarrow \sqrt{1 + y^2} = - \int \left(1 + \frac{1}{v^2 - 1} \right) dv$$

$$\Rightarrow \sqrt{1 + y^2} = -v - \frac{1}{2} \log \left| \frac{v - 1}{v + 1} \right| + C$$

$$\Rightarrow \sqrt{1 + y^2} + \sqrt{1 + x^2} + \frac{1}{2} \log \left| \frac{\sqrt{1 + x^2} - 1}{\sqrt{1 + x^2} + 1} \right| = C$$

36. (i) The given differential equation can be written

as $\frac{dy}{dx} + \frac{\cos x}{1 + \sin x} y = \frac{-x}{1 + \sin x}$

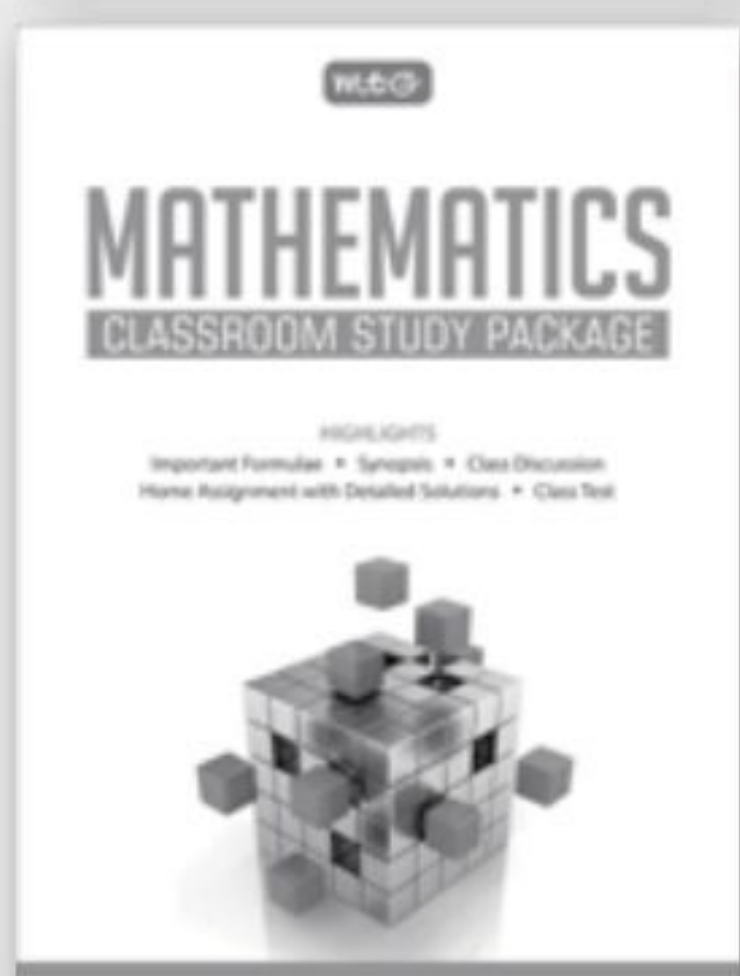
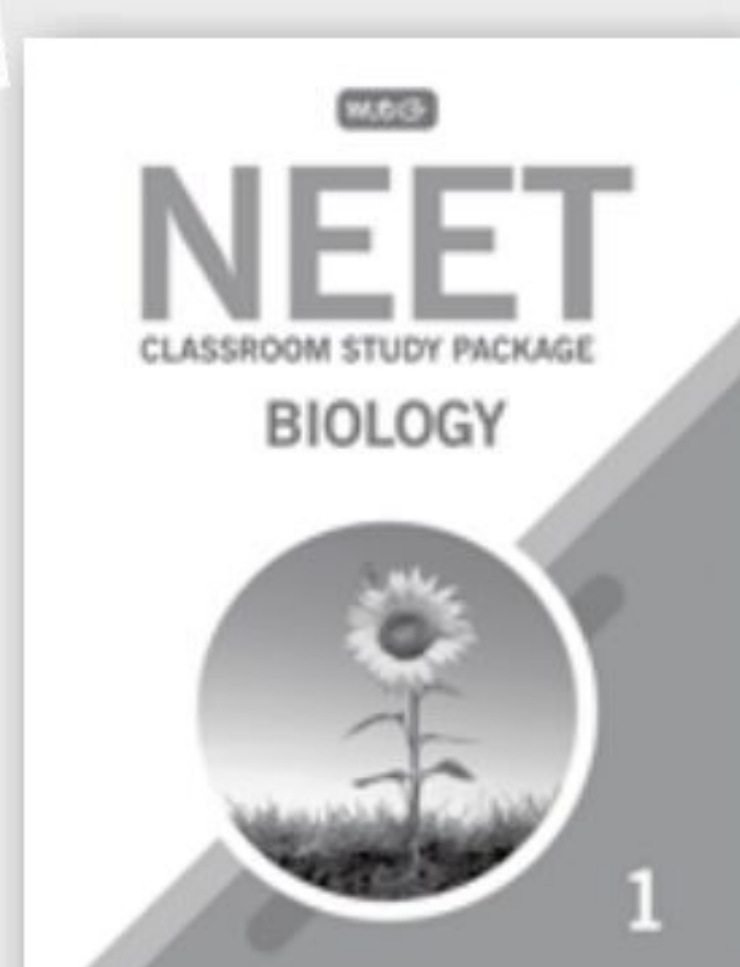
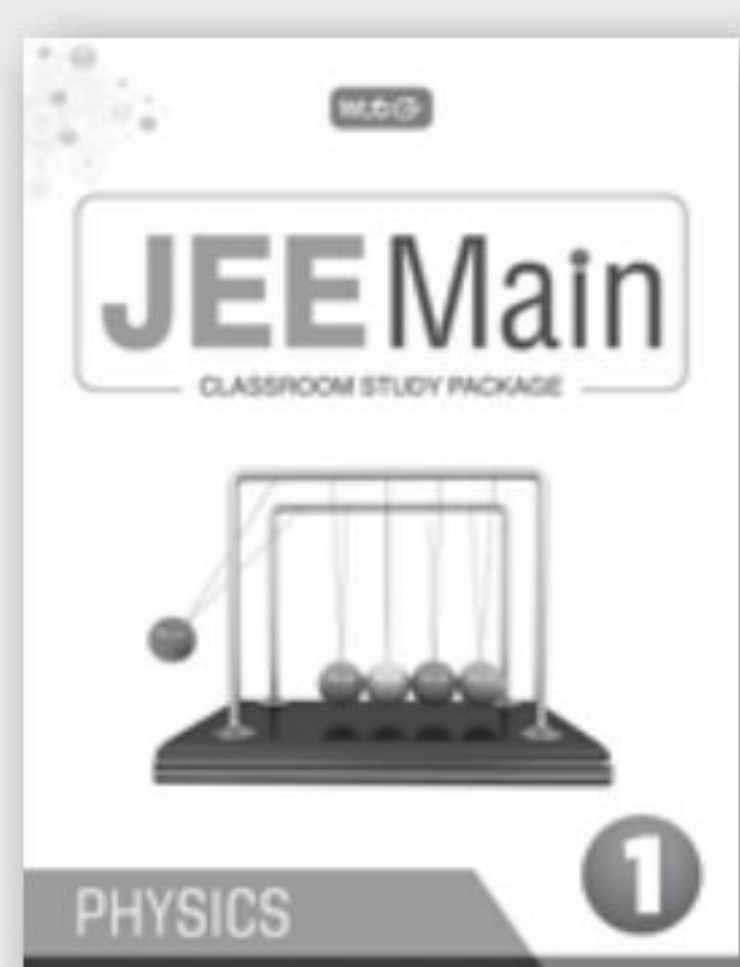
Comparing with $\frac{dy}{dx} + Py = Q$

$$P = \frac{\cos x}{1 + \sin x}, Q = \frac{-x}{1 + \sin x}$$

**ATTENTION
COACHING
INSTITUTES :**

a great offer from
MTG

CLASSROOM STUDY MATERIAL



MTG offers “**Classroom Study Material**” for JEE (Main & Advanced), NEET and FOUNDATION MATERIAL for Class 6, 7, 8, 9, 10, 11 & 12 **with YOUR BRAND NAME & COVER DESIGN.**

This study material will save your lots of money spent on teachers, typing, proof-reading and printing. Also, you will save enormous time. Normally, a good study material takes 2 years to develop. But you can have the material printed with your logo delivered at your doorstep.

Profit from associating with MTG Brand – the most popular name in educational publishing for JEE (Main & Advanced)/NEET....

Order sample chapters on Phone/Fax/e-mail.

Phone : 0124-6601200

e-mail : sales@mtg.in | www.mtg.in



Integrating factor = $e^{\int P dx} = e^{\int \frac{\cos x}{1+\sin x} dx}$

Put $1 + \sin x = t$, $\cos x dx = dt$

$$\therefore \text{I.F.} = e^{\int \frac{1}{t} dt} = e^{\log t} = t = 1 + \sin x$$

(ii) General solution of given differential equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + C$$

$$y(1 + \sin x) = \int \frac{-x}{1 + \sin x} (1 + \sin x) dx + C$$

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + C$$

(iii) We have, $y(0) = 1$ i.e. $x = 0$, $y = 1$

$$\therefore (1 + \sin 0) = C \Rightarrow C = 1$$

$$\therefore y(1 + \sin x) = -\frac{x^2}{2} + 1 = \frac{2 - x^2}{2}$$

$$\therefore y = \frac{2 - x^2}{2(1 + \sin x)}$$

OR

$$\text{We have } y = \frac{2 - x^2}{1 + \sin x}$$

$$y\left(\frac{\pi}{2}\right) = \frac{2 - \left(\frac{\pi}{2}\right)^2}{2\left(1 + \sin \frac{\pi}{2}\right)} = \frac{2 - \frac{\pi^2}{4}}{4} = \frac{8 - \pi^2}{16}$$

37. (i) Given, T is the temperature of the body at any time t . Then, by Newton's law of cooling, we get

$$\frac{dT}{dt} = k(T - 50), \text{ where } k \text{ is the constant of proportionality.}$$

$$\therefore f(T) = T - 50$$

$$(ii) \frac{dT}{dt} = k(T - 50) \Rightarrow \frac{dT}{T - 50} = k dt$$

On integrating both sides, we get

$$\log|T - 50| = kt + \log C \Rightarrow T - 50 = Ce^{kt}$$

Clearly, for $t = 0$, $T = 70^\circ \Rightarrow C = 20$. Thus, $T - 50 = 20e^{kt}$

For $t = 2$, $T = 60^\circ \Rightarrow 10 = 20e^{2k}$

$$\Rightarrow 2k = \log\left(\frac{1}{2}\right) \Rightarrow k = \frac{1}{2} \log\left(\frac{1}{2}\right)$$

$$\text{Hence, } T = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$$

$$(iii) \text{ We have, } T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$$

$$\text{Now, } 98.6 = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$$

$$\Rightarrow \frac{48.6}{20} = \left(\frac{1}{2}\right)^{\frac{t}{2}} \Rightarrow \log\left(\frac{48.6}{20}\right) = \frac{t}{2} \log\left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{t}{2} = \frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)} \Rightarrow t = 2 \left(\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)} \right) \approx -2.56$$

So, it appears that the person was murdered 2.5 hours before 8 p.m. i.e., about 5 : 30 p.m.

OR

$$\text{Since } T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$$

$$\lim_{t \rightarrow \infty} T = \lim_{t \rightarrow \infty} 50 + 20\left(\frac{1}{2}\right)^{t/2} = 50 + 0 = 50$$

$$38. (i) \text{ We have, } \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

$$\text{Put } y = vx \text{ and } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{x^2 + x \cdot vx + v^2 x^2}{x^2} = 1 + v + v^2$$

$$\Rightarrow x \frac{dv}{dx} = 1 + v^2 \Rightarrow \int \frac{dv}{1 + v^2} = \int \frac{dx}{x}$$

$$\Rightarrow \tan^{-1} v = \log|x| + C \Rightarrow \tan^{-1} \frac{y}{x} = \log|x| + C$$

$$(ii) \text{ We have, } 2xy \frac{dy}{dx} = x^2 + 3y^2 \Rightarrow \frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\text{Put } y = vx \text{ and } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{x^2 + 3v^2 x^2}{2vx^2} \Rightarrow x \frac{dv}{dx} = \frac{1 + 3v^2}{2v} - v$$

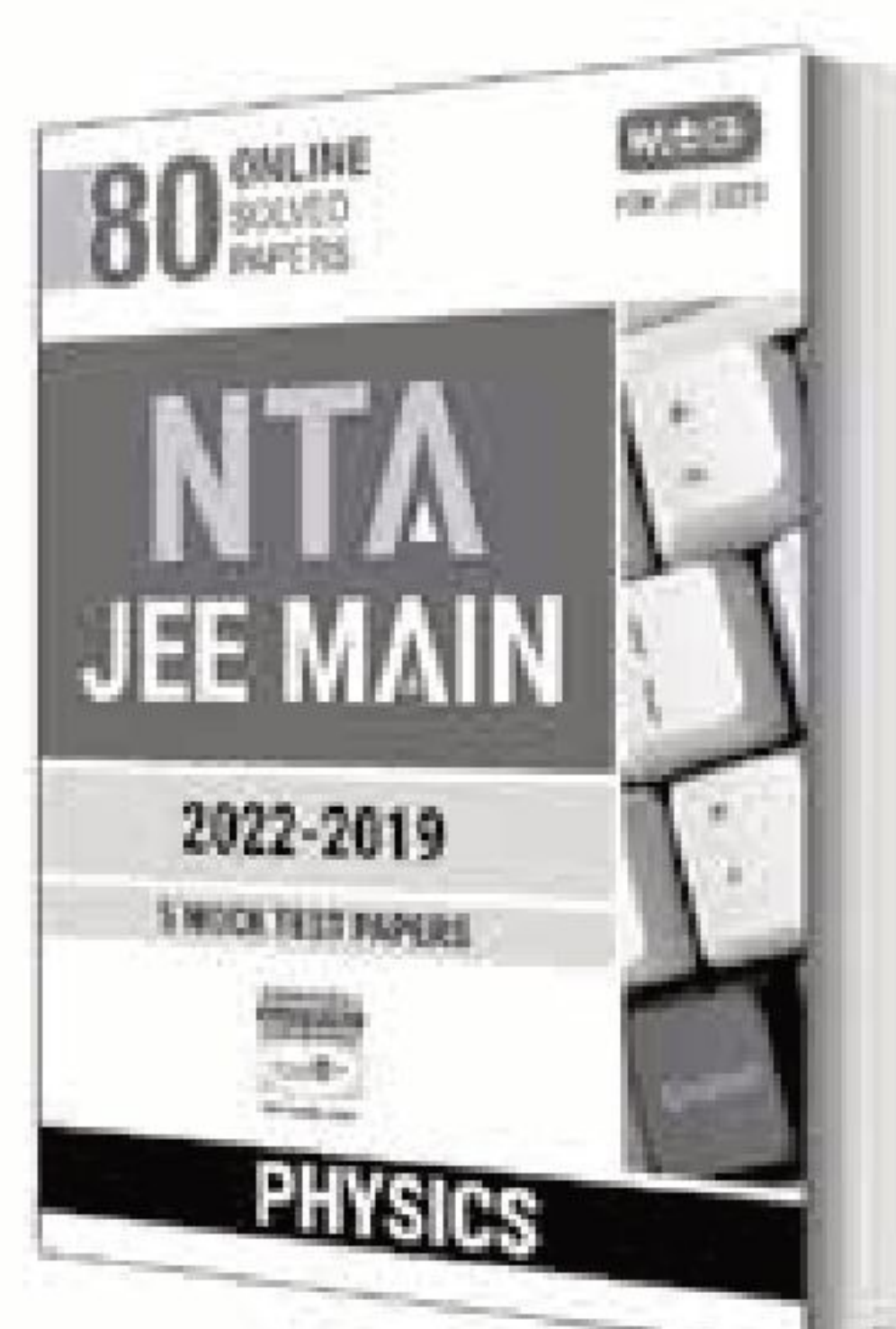
$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{2v} \Rightarrow \int \frac{2v}{1 + v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \log|1 + v^2| = \log|x| + \log|C| \Rightarrow \log|v^2 + 1| = \log|xC|$$

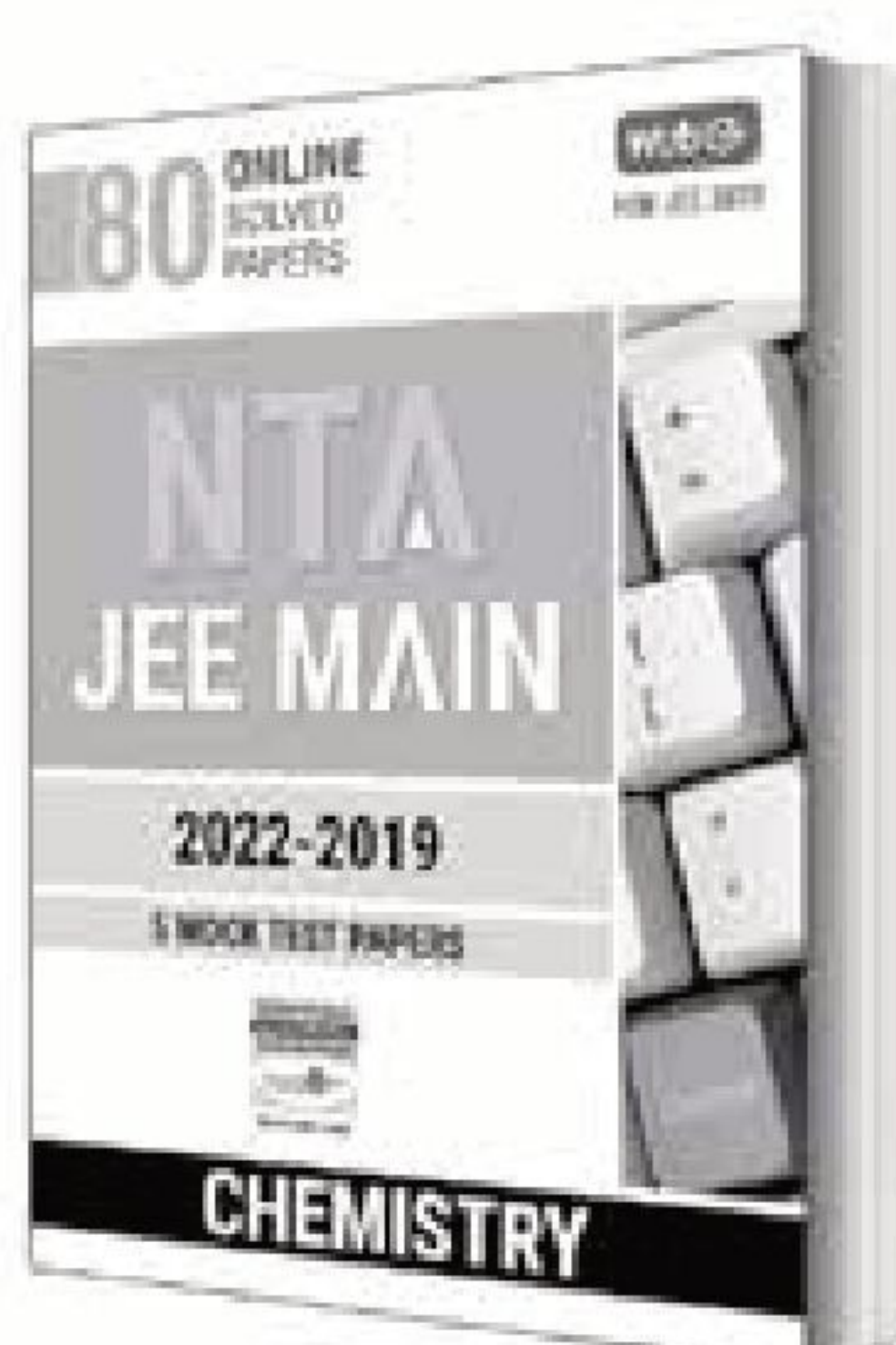
$$\Rightarrow v^2 + 1 = xC \Rightarrow \frac{y^2}{x^2} + 1 = xC \Rightarrow x^2 + y^2 = x^3 C$$



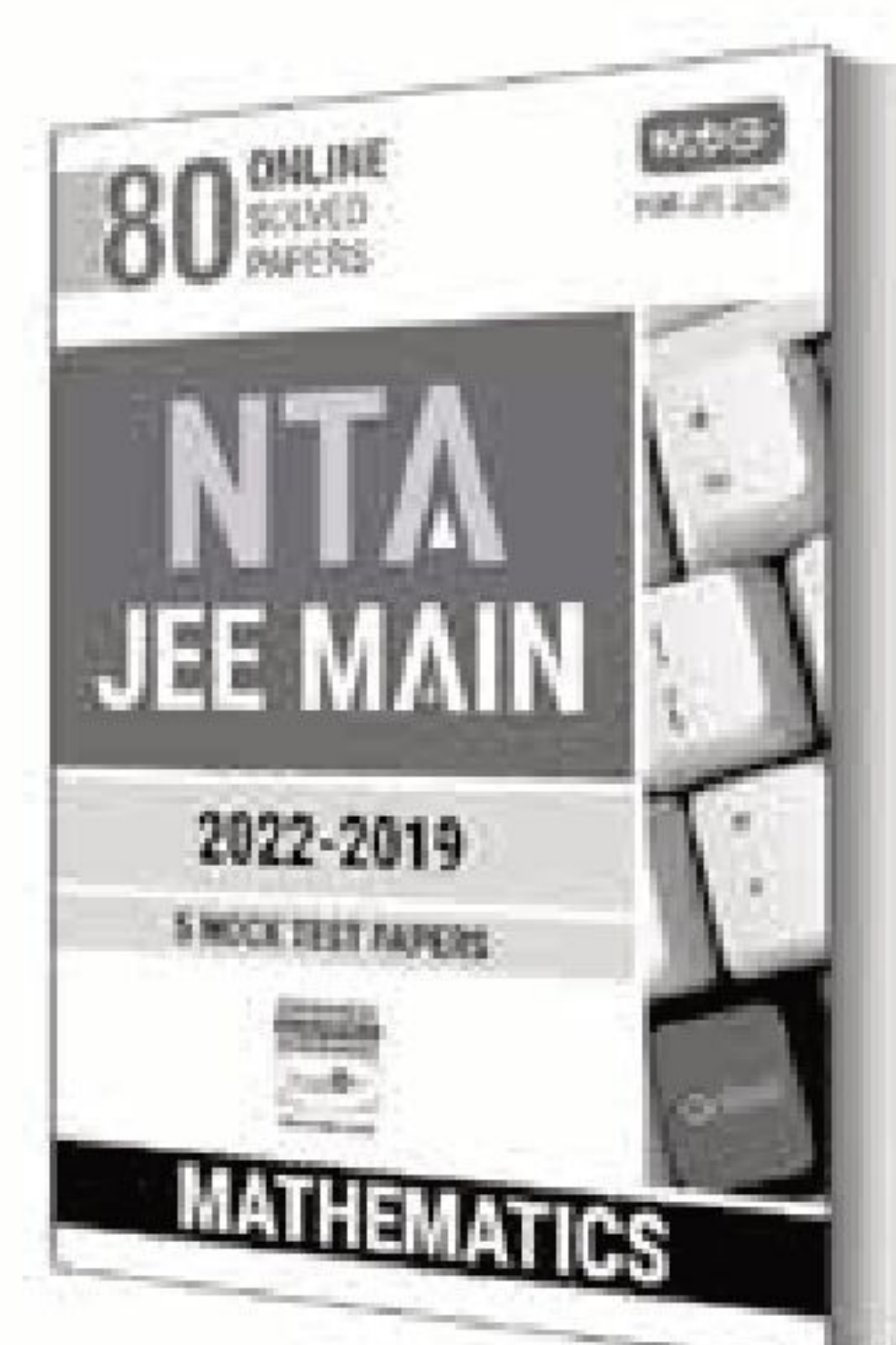
Last Minute Check on your JEE Readiness



₹ 675/-



₹ 675/-



₹ 675/-

The Wait is Over! MTG Presents you all the 80 Fully Solved online Question Papers of JEE Main of past 4 years (2022-2019) conducted by NTA with Chapterwise graphical analysis showing the weightage of chapters and Mock Test Papers as per latest pattern of JEE Main. Get the maximum benefit from the book through personalised course on **Pedagogy app**. The best preparation for tomorrow is doing your best today so, go and get your copy.

Highlights

- Fully Solved Authentic 80 Online Papers
- 6895 Questions for Practice
- Chapterwise Tabular and Graphical Analysis Showing the Weightage of Chapters
- 5 MTPs as per the latest JEE Main pattern

Practice Online Tests to Predict Your Rank



Make It Happen, CRACK JEE Main

Get the complete feel of Online exam
by attempting tests on computers



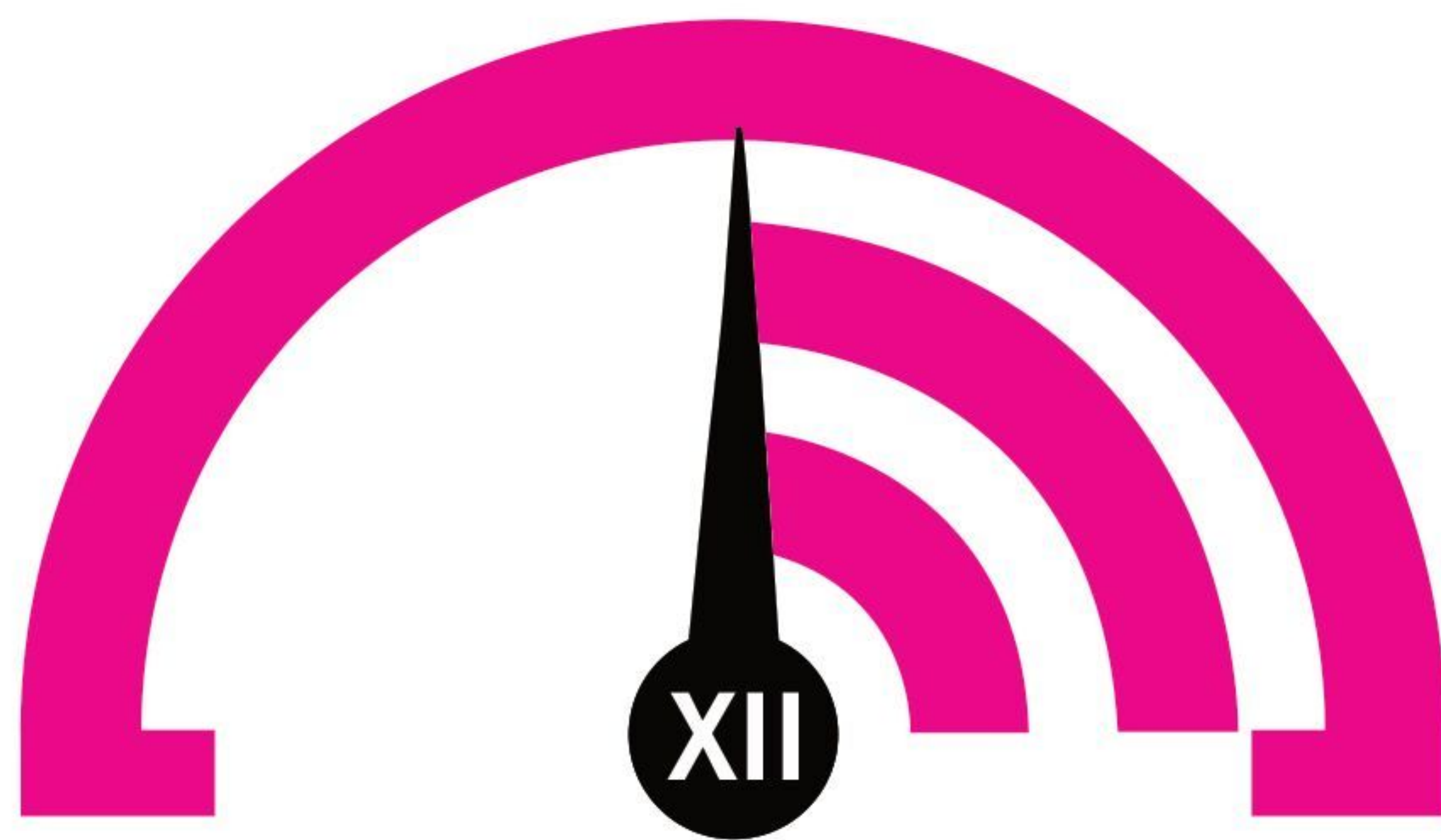
Take the Final Leap Towards *JEE Main* Success!

Available at all leading book shops throughout the country. To buy online visit www.mtg.in.
For more information or for help in placing your order,
Call 0124-6601200 or email: info@mtg.in



Scan to explore & buy more
JEE Books on mtg.in

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 5 : Application of Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

- The normal to the curve $x = a(1 + \cos \theta)$, $y = a \sin \theta$ at θ always passes through the point
(a) $(a, 0)$ (b) $(0, a)$ (c) (a, a) (d) $(0, 0)$
- Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. If two of the normals are perpendicular to each other, then $c =$
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1
- The points on the curve $y^3 + 3x^2 = 12y$ where tangent is vertical, are
(a) $\left(\pm \frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm \frac{4}{\sqrt{3}}, 2\right)$
(c) $(0, 0)$ (d) $\left(\pm \frac{11}{3}, 0\right)$
- The curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$ intersect at an angle of
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$
- If tangents are drawn from the origin to the curve $y = \sin x$, then their points of contact lie on the curve
(a) $x - y = xy$ (b) $x + y = xy$
(c) $x^2 - y^2 = x^2y^2$ (d) $x^2 + y^2 = x^2y^2$
- The slope of the straight line which is both tangent and normal to the curve $4x^3 = 27y^2$ is
(a) ± 1 (b) $\pm \frac{1}{2}$ (c) $\pm \frac{1}{\sqrt{2}}$ (d) $\pm \sqrt{2}$

One or More Than One Option(s) Correct Type

- If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then
(a) $a > 0, b > 0$ (b) $a > 0, b < 0$
(c) $a < 0, b > 0$ (d) $a < 0, b < 0$
- The function $f(x) = 2x^3 - 3(\lambda + 2)x^2 + 2\lambda x + 5$ has one maximum and one minimum if $\lambda \in$
(a) $(-4, \infty)$ (b) $(-\infty, 0)$
(c) $(-3, 3)$ (d) $(1, \infty)$



ONLINE TEST SERIES

Practice Part Syllabus/ Full Syllabus
24 Mock Tests for

JEE Main



Now on your android smart phones
with the same login of web portal.

Log on to test.pcmbtoday.com

9. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes, (where $\alpha^2 + \beta^2 = 61$) then the value of a is
(a) -30 (b) 10 (c) 20 (d) 30

10. Let $f''(x) > 0 \forall x \in R$ and $g(x) = 2f\left(\frac{x^2}{2}\right) + f(6 - x^2)$.

Then

- (a) g attains maximum at $x = \pm 1$
(b) g attains minimum at $x = \pm 1$
(c) g attains maximum at $x = 0$
(d) g attains minimum at $x = \pm 2$
11. Let $f: R \rightarrow (0, \infty)$ and $g: R \rightarrow R$ be twice differentiable functions such that f'' and g'' are continuous functions on R . Suppose $f'(2) = g(2) = 0, f''(2) \neq 0$ and $g'(2) \neq 0$. If $\lim_{x \rightarrow 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then
(a) f has a local minimum at $x = 2$
(b) f has a local maximum at $x = 2$
(c) $f''(2) > f(2)$
(d) $f(x) - f''(x) = 0$ for at least one $x \in R$
12. Let $f(x)$ be a differentiable function and $f(a) = f(b) = 0$ ($a < b$), then which of the following can be true in the interval (a, b) ?
(a) $f(x) + f'(x) = 0$ has at least one root.
(b) $f(x) - f'(x) = 0$ has at least one root.
(c) $f(x) \cdot f'(x) = 0$ has at least one root.
(d) none of these

13. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is
(a) increasing on $[0, \infty)$
(b) decreasing on $[0, \infty)$
(c) increasing on $[0, \pi/e)$ and decreasing on $[\pi/e, \infty)$
(d) decreasing on $[0, \pi/e)$ and increasing on $[\pi/e, \infty)$

Comprehension Type

Paragraph for Q. No. 14 and 15

Consider $f(x) = |x - 1| + |x - 3| + |5 - x|, \forall x \in R$.

14. If $f(x)$ increases, then $x \in$
(a) $(1, \infty)$ (b) $(3, \infty)$ (c) $(5, \infty)$ (d) $(1, 3)$

15. If $f(x)$ decreases, then $x \in$
(a) $(-\infty, 1)$ (b) $(-\infty, 3)$ (c) $(-\infty, 5)$ (d) $(3, 5)$

Matrix Match Type

16. Match the following :

Column-I		Column-II	
(P)	$f(x) = \cos \pi x + 10x + 3x^2 + x^3, -2 \leq x \leq 3$. The absolute minimum value of $f(x)$ is	(1)	$3/4$
(Q)	If $x \in [-1, 1]$, then the minimum value of $f(x) = x^2 + x + 1$, is	(2)	2
(R)	Let $f(x) = (4/3)x^3 - 4x, 0 \leq x \leq 2$. Then, the global minimum value of the function is	(3)	-15
(S)	Let $f(x) = 6 - 12x + 9x^2 - 2x^3, 1 \leq x \leq 4$. Then the absolute maximum value of $f(x)$ in the interval is	(4)	$-8/3$

	P	Q	R	S
(a)	2	1	3	4
(b)	3	2	1	4
(c)	3	1	4	2
(d)	1	2	3	4

Numerical Answer Type

17. The maximum slope of the curve $-x^3 + 3x^2 + 2x - 27$ is _____.
18. The area of the largest circle with centre $\left(\frac{1}{2}, 0\right)$ inscribed in the ellipse $x^2 + 2y^2 = 2$ is $\frac{m\pi}{n}$, where m and n are relatively prime positive integers. Then $m + n$ is equal to _____.
19. The maximum possible integral value of $\frac{\beta - \alpha}{\tan^{-1} \beta - \tan^{-1} \alpha}$, where $0 < \alpha < \beta < \sqrt{3}$, is _____.
20. If $y = a \ln x + bx^2 + x$ has its extreme values at $x = -1, 2$, then $2(a + b)$ is equal to _____.



Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted
No. of questions correct
Marks scored in percentage

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

mtg

SUBSCRIBE NOW

SAVE ₹ 920/-*
with subscription plans

Get them uninterruptedly

SUBSCRIPTION FORM

☐ Student ☐ Teacher

Name: _____

Complete Postal Address: _____

Pin Code: _____ Mobile No.: _____

Email: _____

Confirm your choice by placing ☒ tick-mark in relevant boxes.

SELECT PLAN:

9 Months ☐ 15 Months ☐ 27 Months ☐

Individual Magazines

- ☐ PHYSICS FOR YOU (P)
☐ CHEMISTRY TODAY (C)
☐ MATHEMATICS TODAY (M)
☐ BIOLOGY TODAY (B)

Get Best Result in Combinations

- ☐ PCB Combo
☐ PCM Combo
☐ PCMB Combo

Subscription Price:

Months	27 Months	15 Months	9 Months
Individual Magazines Physics Chemistry Mathematics Biology	₹ 850/- (Save ₹ 230/-)	₹ 500/- (Save ₹ 100/-)	₹ 300/- (Save ₹ 60/-)
Combo of 3 Magazines PCM PCB	₹ 2500/- (Save ₹ 740/-)	₹ 1400/- (Save ₹ 400/-)	₹ 900/- (Save ₹ 180/-)
Combo of 4 Magazines PCMB	₹ 3400/- (Save ₹ 920/-)	₹ 1900/- (Save ₹ 500/-)	₹ 1200/- (Save ₹ 240/-)
Courier Charges Add to your subscription amount for quicker & reliable delivery	₹ 600/-	₹ 450/-	₹ 240/-

Note: Magazines are dispatched by Book-Post on 4th of every month (each magazine separately)

Recommended by (optional)

Name of your teacher: _____

Teacher's Mobile No.: _____

*on cover price

Enclose Demand Draft favouring
MTG Learning Media (P) Ltd. payable at New Delhi.
Mail this Subscription Form to Subscription Department.

MTG Learning Media (P) Ltd., Plot 99, Sector 44, Gurugram-122 003 (HR)

Get Digital Editions of MTG Magazines on <http://digital.mtg.in/>



subscription@mtg.in



www.mtg.in
To subscribe online

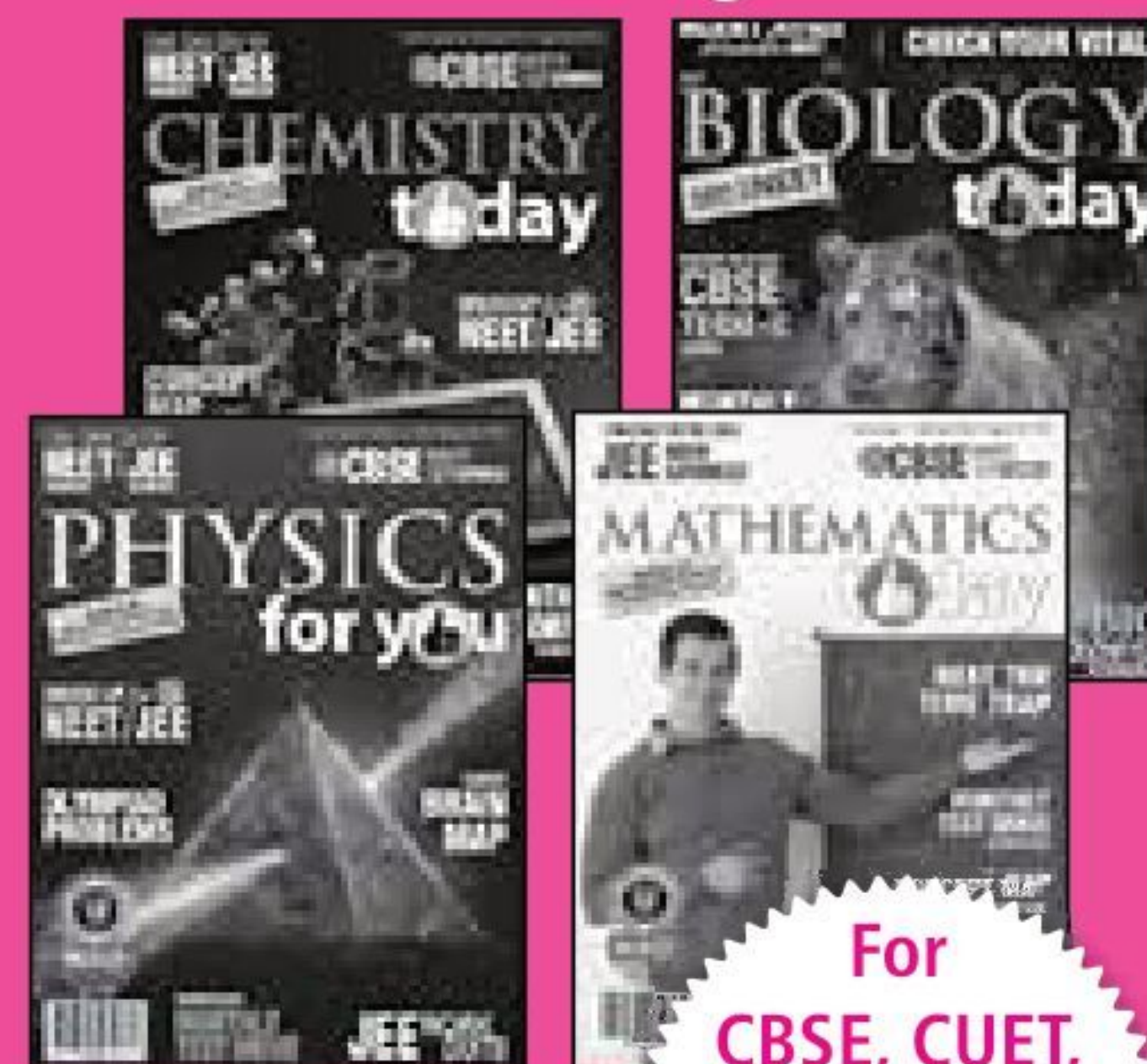


0124-6601200



Trust of Millions, Since 1982

www.mtg.in



@40/- each
80 pages in each

For
CBSE, CUET,
NEET, JEE &
OTHER PET
EXAMS

WHY? MTG MAGAZINES

Not every student can afford the hefty fees of coaching institutes yet dreams to clear NEET or JEE or CBSE or CUET exams with excellent marks. MTG makes it easy and affordable for all students to access world class content created by top faculties & experts across the globe through its Monthly Magazines and Books.

In these magazines you will get

- ✓ Quality Practice papers
 - ✓ Concept Explanations
 - ✓ Solved Papers
 - ✓ Current Updates
 - ✓ Fun Educational Activities
- ...and much more.



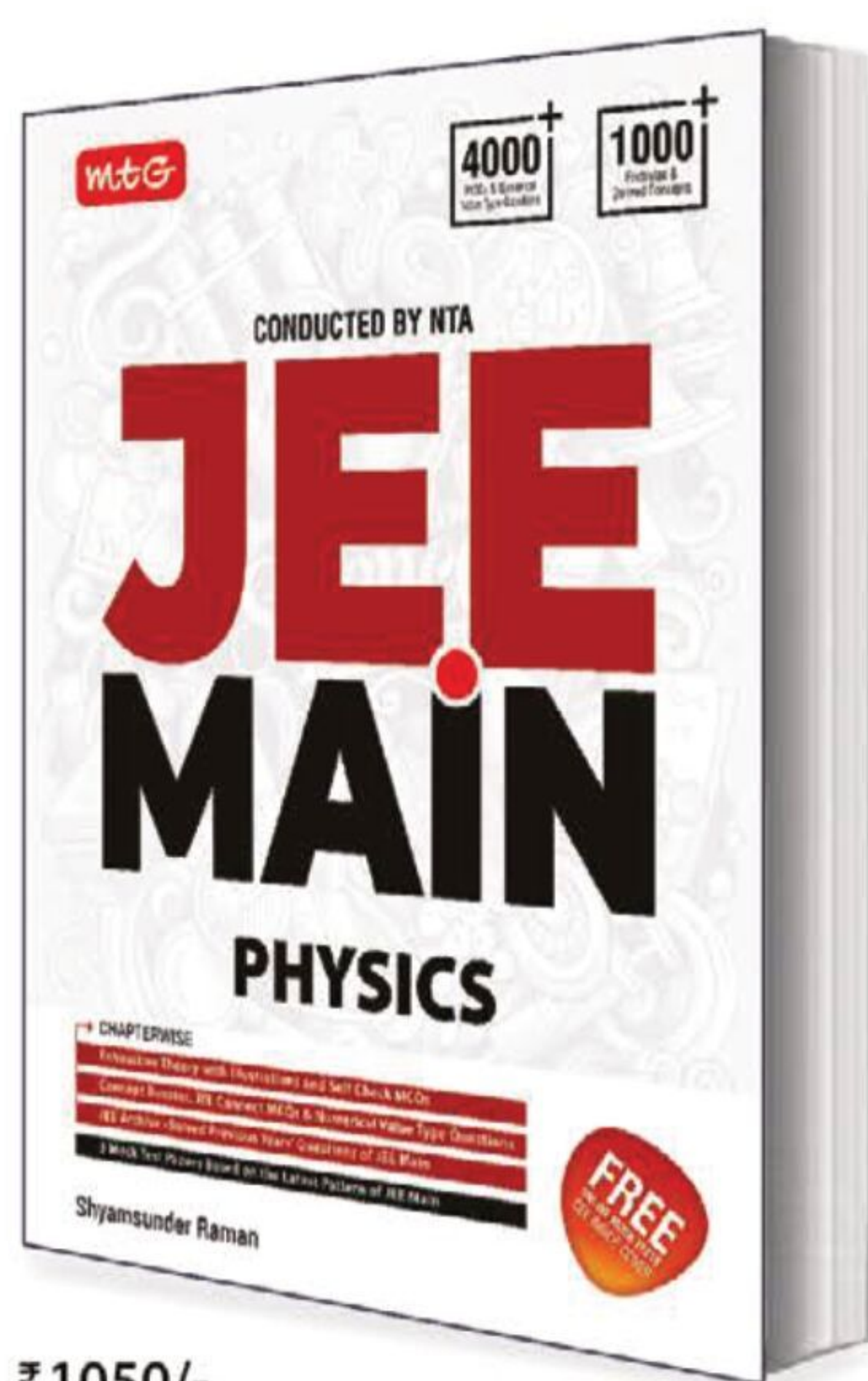
LIFETIME SUBSCRIPTION

Plan for teachers and special schemes and offer available for libraries and coaching institutes.
SMS MTG to 8800255334 to learn more.

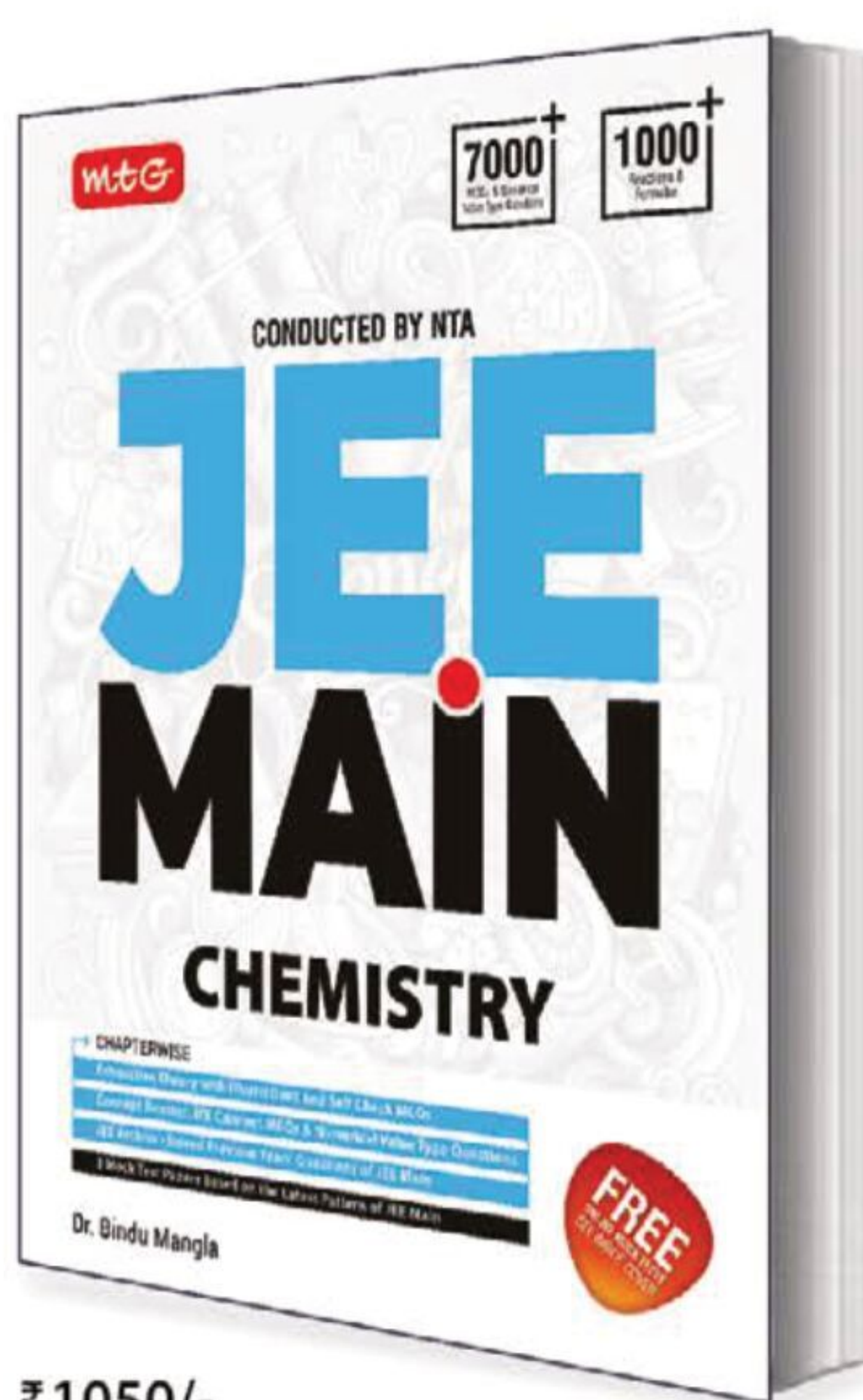
Study right. Dig deep.

mtg

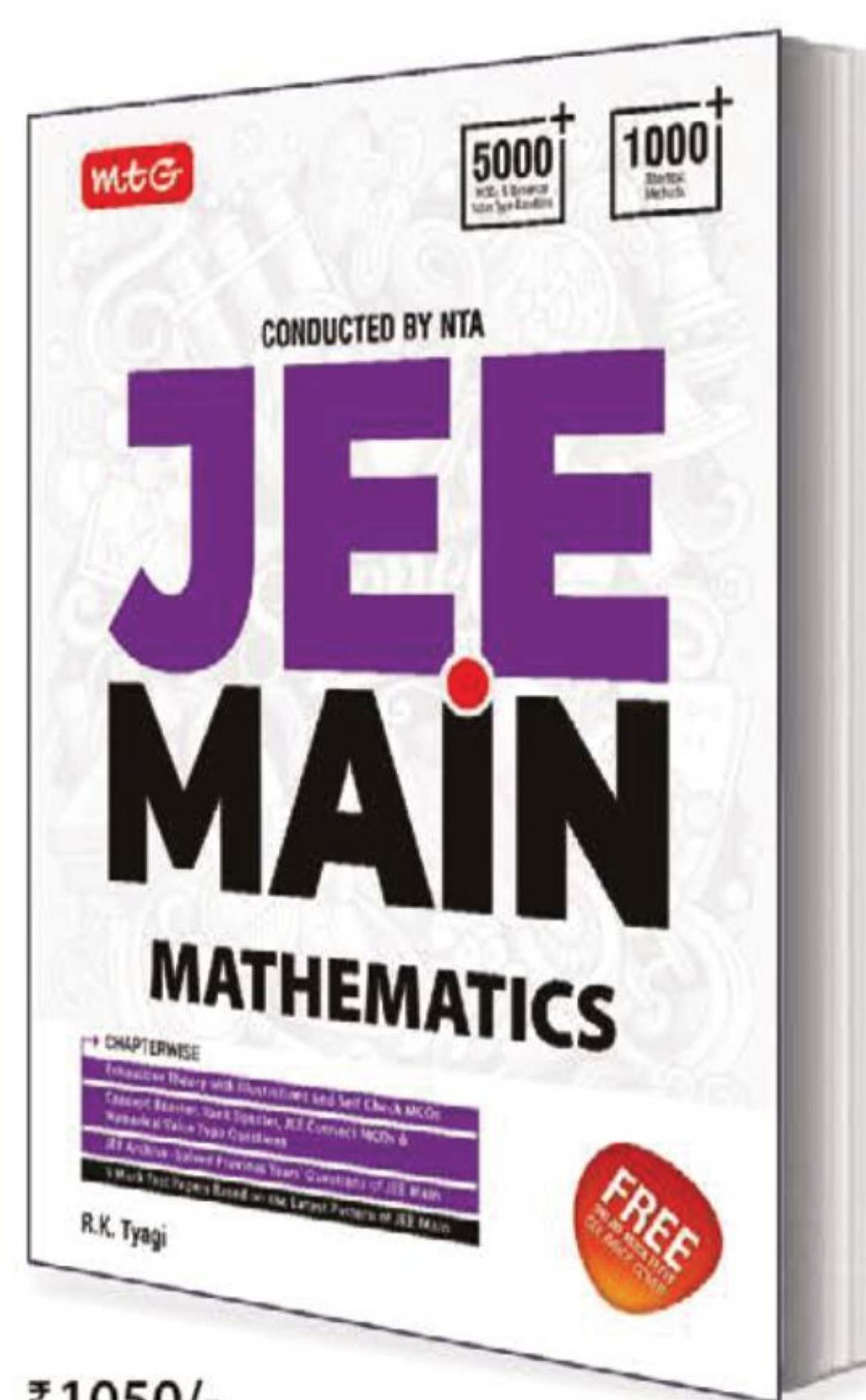
Build a solid foundation for success
in JEE Main



₹ 1050/-



₹ 1050/-



₹ 1050/-

Are you a do-it-yourself type of a student? Then for success in JEE Main, choose MTG's JEE Main combo, comprising coursebooks for Physics, Chemistry & Mathematics. This combo is all class XI and XII students need for a solid and deep understanding of concepts in these three key subjects.

FEATURES

- Based on latest pattern of JEE Main
- Covers the entire syllabus
- Full of graphic illustrations for deep understanding of concepts
- Exhaustive Theory with Illustrations and Self Check MCQs
- Chapterwise Concept Booster, Rank Booster and JEE Connect MCQs
- Chapterwise Numerical Value Type Questions
- JEE Archive-Solved Previous Years' Questions of JEE Main
- 3 Mock Test Papers based on Latest Pattern of JEE Main



Scan now with your
smartphone or tablet
Application to read
QR codes required

Note: Coursebooks are also available separately.

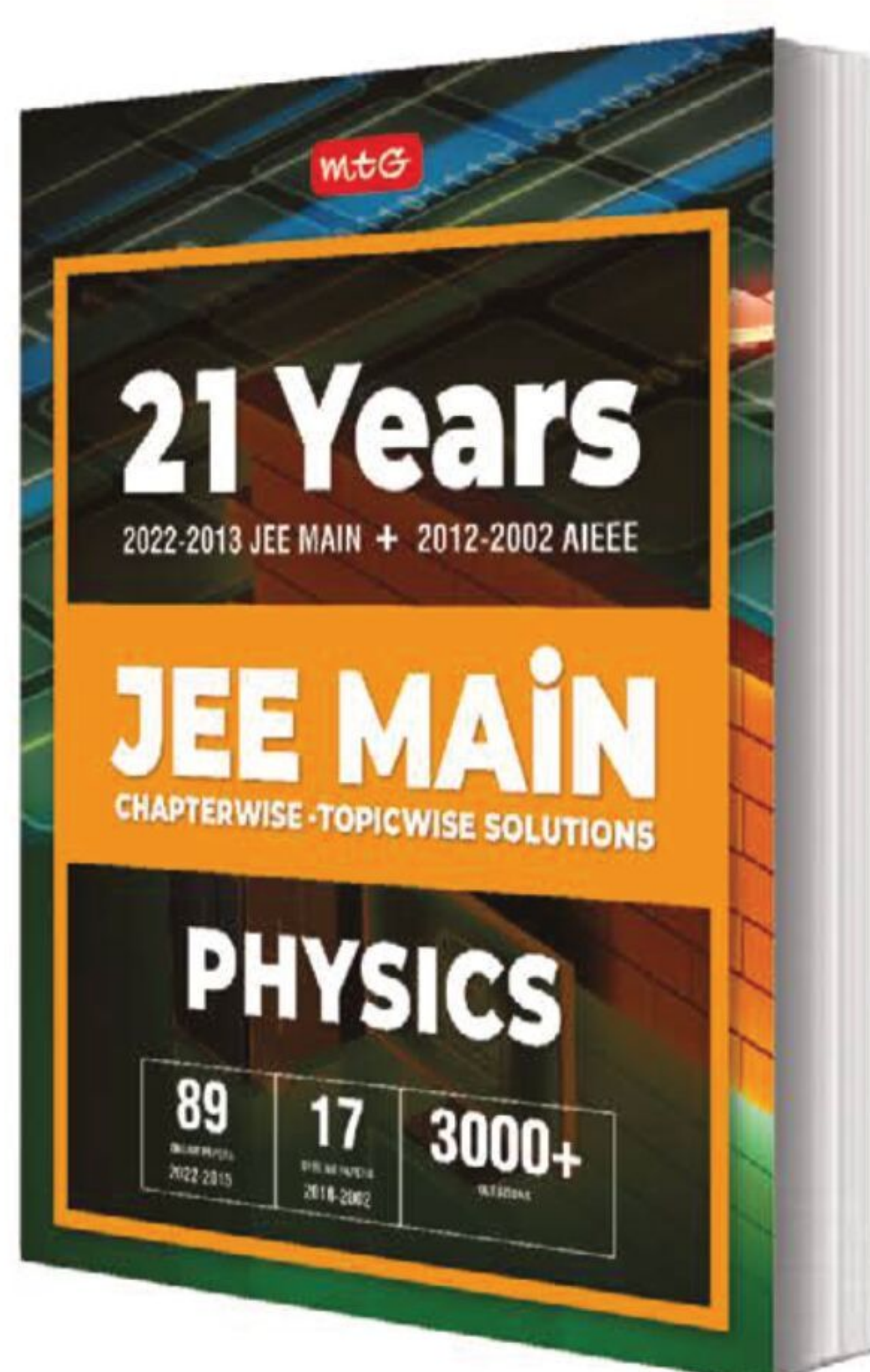
Available at all leading book shops throughout India. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or e-mail: info@mtg.in

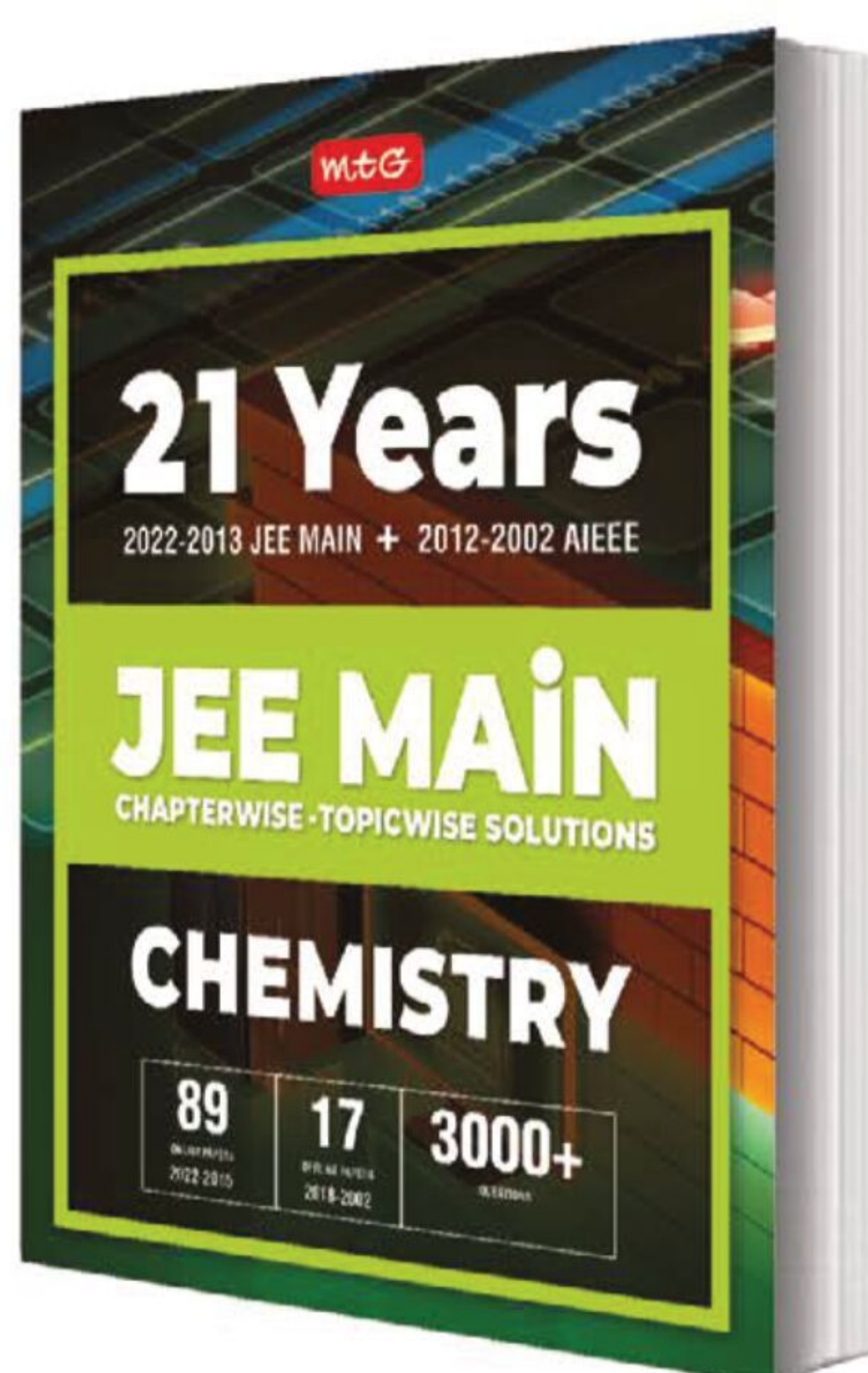


The Best tool for success in JEE Main

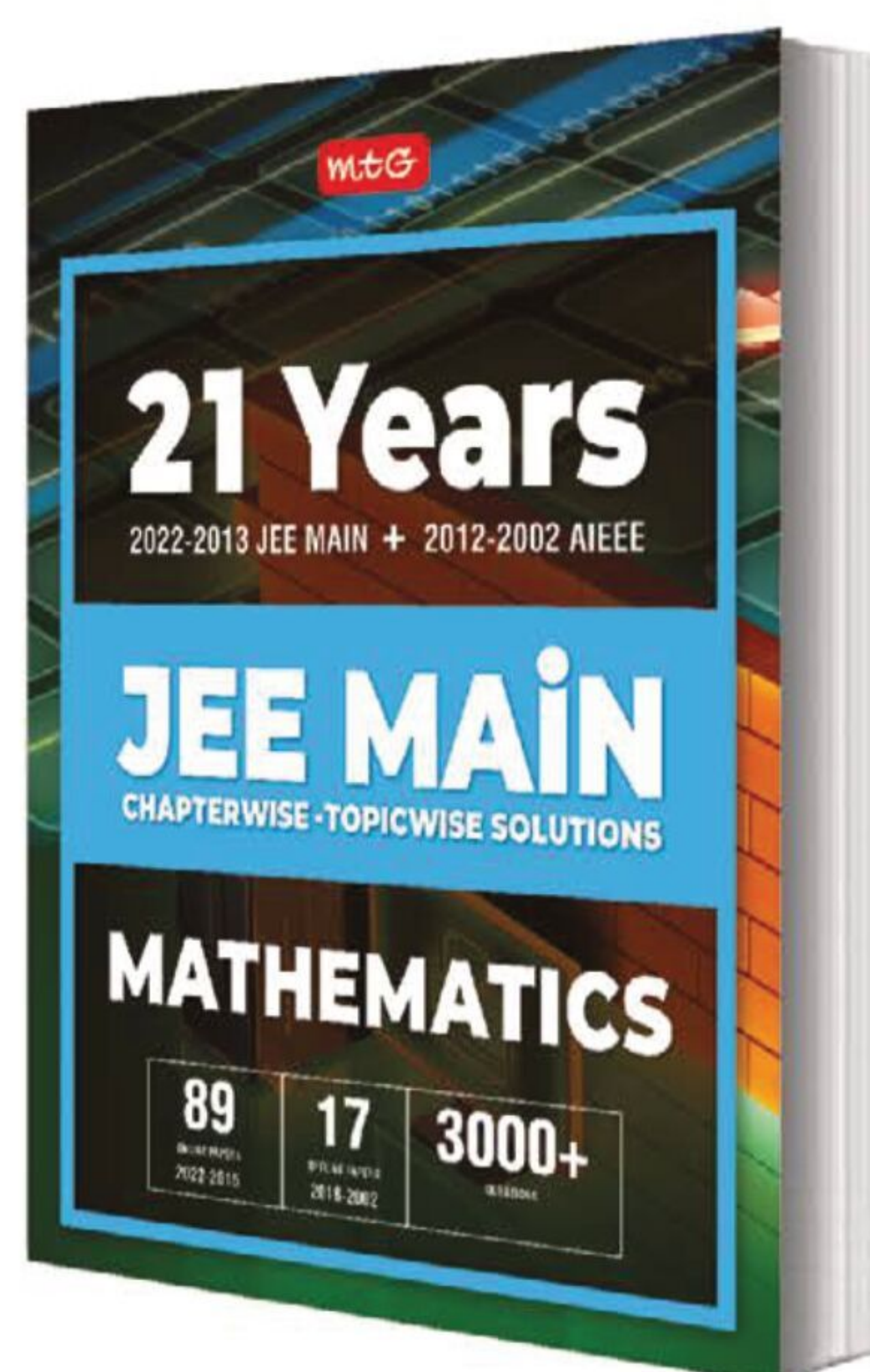
21 Years' JEE Main Chapterwise-Topicwise Solutions Physics, Chemistry and Mathematics contain not only chapterwise - topicwise questions that have appeared over the last 21 years in JEE Main / AIEEE but also their complete solutions. Needless to say these books are essential for any student to compete successfully in JEE Main.



₹ 800/-



₹ 800/-



₹ 800/-

HIGHLIGHTS:

- Chapterwise-Topicwise Questions (Online & Offline) of last 21 years' (2022-2002) of JEE Main / AIEEE
- Chapterwise-Topicwise segregation of questions to help you assess the level of effort required to succeed
- An unmatched series with more than 1740 pages filled with detailed solutions by experts

Visit
www.mtg.in
for latest offers
and to buy
online!



MTG Learning Media (P) Ltd.
Plot # 99, Sector 44, Gurugram – 122 003 (HR)

Available at all leading book shops throughout India.
For more information or for help in placing your order,
Call 0124-6601200 or e-mail: info@mtg.in



Scan to explore & buy more
JEE Books on mtg.in



How to choose the right answer, fast?



The answer is practice...

Our team has seen that in NEET and JEE, Multiple Choice Questions (MCQs) are based on the NCERT syllabus. Largely !! With Objective NCERT at your FINGERTIPS, you can become a pro at handling MCQs. Practice to increase your accuracy and improve timing with a bank of approx. 15,000 questions, all framed from NCERT course books. Don't take our word, have a look what some of our readers / toppers have to say...

Features:

- NCERT Notes with High Definition (HD) pages
- Topicwise NCERT Based MCQs
- NCERT Exemplar Problems & HOTS MCQs
- Exam Archives-Previous Years' questions
- 6 Practice Papers | Model Test Papers

Kalpna Kumari, "I have followed MTG NCERT at Your Fingertips, Assertion and Reason books and magazines Physics For You, Biology Today and Chemistry Today. In my opinion, full dedication and smartly planning your preparation are the most important components of an ideal preparation."

Mrinal Dewangan, "MTG Objective NCERT at your Fingertips is a great book and Explorers provided very accurate question papers similar to those asked in exams, which helped me a lot during preparation."

Het Shah, "For MCQs I referred to 'MTG Objective NCERT at your Fingertips' which is an exceptional book in itself. Also I referred to all the MTG Books for previous years' question papers of different exams."

Tejaswin Jha, "The mantra is to study NCERT books rather than studying those heavy weight expensive materials. NCERT is the best way to clear your base and concepts in all the three subjects – Physics, Chemistry and Biology. Once your base is clear, scoring in AIPMT is easier for anyone."

Lovedeep Singh Dhingra, "I found NCERT books to be very helpful, especially Biology and Chemistry. Most of the Biology section was from NCERT book. Around 95% Biology section in AIPMT exam come from NCERT books only."

Arnav Aggarwal, "AIIMS does not declare score; they only announce ranks. As for the study plan is concerned, I did not really have to work hard. There were some topics that were not covered in NEET but were included in NCERT; so I studied that."



Scan now with your smartphone or tablet

Application to read QR codes required



MTG Learning Media (P) Ltd.
Plot #99, Sector 44, Gurugram – 122 003 (HR)

Available at all leading book shops throughout India.

For more information or for help in placing your order,
Call 0124-6601200 or e-mail: info@mtg.in

> Aakashians Champions of NEET / JEE 2022



>> NEET-UG 2022

83440

70339 Classroom + 13101 Distance & Digital

**Aakashians
Qualified in
NEET-UG 2022**

AIR
2



Vatsa A Batra
2 Year Classroom

AIR
3



Hrishikesh N G
2 Year Classroom

AIR
9



Zeel Vipul Vyas
4 Year Classroom

AIR
10



Haziq Parveez L
2 Year Classroom

AIR
11



Sayantani Chatterjee
2 Year Classroom

and many more...

>> JEE (Advanced) 2022

1898

1588 Classroom + 310 Distance & Digital

**Aakashians
Qualified in
JEE (Adv.) 2022**



Tanishka Kabra
4 Year Classroom



Jaladhi Joshi
5 Year Classroom

AIR
91



Ananya Rao
2 Year Classroom

AIR
99



Hemanshu Garg
5 Year Classroom

AIR
136



Apoorv Tandon
2 Year Classroom

and many more...

>> JEE (Main) 2022

8455

7181 Classroom + 1274 Distance & Digital

**Aakashians
Qualified in
JEE (Main) 2022**

AIR
35



Hemanshu Garg
5 Year Classroom

AIR
61



Jaladhi Joshi
5 Year Classroom

AIR
71



Apoorv Tandon
2 Year Classroom

AIR
125



Tarun
1 Year Classroom

AIR
132



Ananya Rao
2 Year Classroom

and many more...

Though every care has been taken to publish the result correctly, yet Aakash Educational Services Ltd. shall not be responsible for inadvertent error, if any.

Scan the QR code to
check 2022 results &
download the
Result Booklet

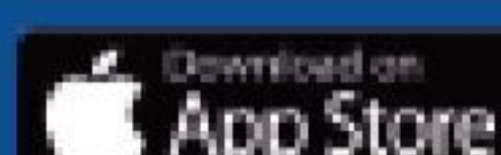
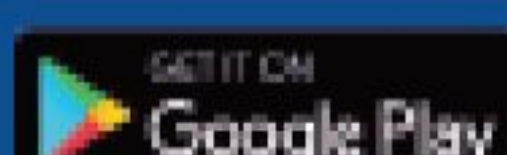


ADMISSIONS OPEN

- > **Medical (NEET)**
- > **Engineering (JEE)**
- > **Foundation (Class 8-10)**



Download **Aakash App**



CALL (TOLL-FREE) **1800-102-2727**

VISIT **aakash.ac.in**